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Relativity and technology

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In this paper we first give a short outline of the principles and experimental status of tests of relativity, which indicates the huge still ongoing progress during the last years. Then emphasize is laid on the fact that relativity, either special or general, already enters a status of an applied technology in daily life. Examples are presented from positioning, metrology, earth sciences, and material sciences.

1 Introduction

Relativistic physics in general is considered as being important for pure science, for the general understanding of the deepest laws underlying our world, governing the evolution of the universe and the dynamics of the smallest elementary particles, the quarks. Within such a point of view, the physics of the big bang or of quarks certainly is of no direct relevance for our daily life.

However, and this is what we like to stress here, relativity in the meantime gained much importance and many applications in technology and its functioning also can be considered as a prerequisite for economical success of some technologies like the Global Positioning System (GPS) and, e.g., based on it, the German toll system on highways which is a market with a flow of many billion Euros per year.

In this article we first state and interpret the principles underlying Special Relativity (SR) and General Relativity (GR) and report shortly the current experimental status of SR and parts of GR. Then we will describe the relativity aspects of (i) the definition of time, that is, of the second on the Earth and in space, (ii) the definition of other physical units, in particular of the metre, (iii) the GPS, (iv) spectrosopy for material sciences, and (v) Earth sciences from tectonic motion to climate research.

2 The basics of relativity

2.1 Special Relativity

2.1.1 The principles

The theory of Special Relativity (SR) is based on two principles, (i) the constancy of the speed of light, and (ii) the independence of experimental results from the state of motion of the laboratory (principle of relativity), see also [1]. The constancy of the speed of light has two aspects, (a) the independence of the speed of light from the velocity of the source, and (b) the independence of the speed of light from the state of motion of the observer, that is, from the velocity or the orientation of the laboratory. The relativity principle is very powerful. It ensures, e.g., that the limiting velocity of all particles like electrons, protons, neutrinos, is the same as the velocity of light, that is, $c = c_+ = c_- = v_e^{\max} = v_p^{\max} = v_\nu = v_{\text{grav}} = \ldots$, where $c_\pm$

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are the velocities of the two polarization states of the electromagnetic radiation and $v_{\rm grav}$ is the velocity of gravitational waves.

2.1.2 The experimental status

The independence of the speed of light from the velocity of the source has been examined astrophysically as well as in laboratory experiments. We mention just two of them: (i) the observation by Brecher [2] who analyzed the light coming from a distant binary system consisting of a dark, heavy central star and an orbiting bright star. We model a hypothetical dependence of the speed of light c from the velocity of the source by $c' = c + \kappa v$, where v is the velocity of the source with respect to the Earth and κ some parameter which is 0 in SR and 1 in Galilean kinematics. If the star is moving away from the Earth, the emitted light is slower than the light emitted when the star is moving towards the Earth (it is reasonable to assume $\kappa > 0$ though the case $\kappa < 0$ can be treated analogously). Therefore, light emitted towards the Earth may overtake the light emitted earlier. The images of the star seen on Earth may show an achronological order. Since this has never been observed one can derive $\kappa \leq 10^{-10}$. A laboratory version of this observation has been carried through at CERN [3]. Protons hitting a Beryllium target created π^0 mesons possessing a velocity of $v = 0.99975c$. These π^0 mesons decay within 10^{-16} s into photons whose velocity has been measured and compared with the velocity of photons emitted from π^0 mesons at rest. No difference in the speed of the photons has been found leading to $\kappa \leq 10^{-6}$. Though this is not as precise as the Brecher result, it shows the result for a velocity of the source being almost the speed of light. There is nothing more convincing than this experiment for the independence of the speed of light from the velocity of the source.

The next step is to examine the equality of all limiting velocities. The equality of the maximum speed of electrons, photons in various frequency ranges, neutrinos and muons has been tested by a variety of experiments, see e.g. [4–7], which all result in a relative equality at the 10^{-6} level. Astrophysical observations of radiation from the supernova SN1987A yield for the comparison of photons and neutrinos an estimate which is two orders of magnitude better. Furthermore, from astrophysical observations one can conclude that the vacuum shows no birefringence up to an order $|(c_+ - c_-)/c_+| \leq 2 \cdot 10^{-32}$ [8].

Due to newly emerged technologies many new tests of the principles of SR, in particular of the three classical tests (isotropy and constancy of the speed of light and time dilation), have recently been carried through with strongly improved accuracy. The relative difference of the speed of light in different directions is now smaller than $\Delta_\vartheta c/c \leq 10^{-16}$ [9–11] and is approaching the 10^{-17} level and the difference of the speed of light in differently moving inertial systems is now smaller than $\Delta_v c/c \leq 10^{-16}$ [12]. These model independent estimates can be converted into estimates of parameters in certain tests theories. Within the kinematical Robertson–Mansouri–Sexl test theory the two–way velocity of light which is involved only in these experiments is expressed as $c(\vartheta, v)/c = 1 + A\frac{v^2}{c^2} + B\frac{v^2}{c^2}\cos^2\vartheta$, where v is the velocity of the laboratory with respect to a preferred frame which usually is taken to be the cosmological frame. In this case the above estimates translate into $|A| \leq 3 \cdot 10^{-7}$ and $|B| \leq 2 \cdot 10^{-10}$ [10]. The experimental results can also be interpreted within a dynamical test theory for the electromagnetic field like the Standard Model Extension [13] or even more general models [14].

Also the time dilation factor has been confirmed with much better accuracy using ions in a storage ring moving with a velocity of $v = 0.064\,c$. With the time dilation factor γ in the parametrization $\gamma(v) = 1 + \left(\frac{1}{2} + \alpha\right)\frac{v^2}{c^2} + \ldots$ (for SR we have $\alpha = 0$) the most recent experiment gave $|\alpha| \leq 2.2. \cdot 10^{-7}$ [15]. Time dilation has also been verified by the decay of moving elementary particles, see [16] for a recent version of these experiments where time dilation has been verified at the 10^{-3} level. Though these experiments are not as precise as the spectroscopic ones they prove the time dilation for a different physical process and, thus, its universality. Another class of time dilation experiments are Moessbauer rotor experiments [17–19]. The achieved accuracy was $|\alpha| \leq 10^{-5}$. It has been claimed [20] that an accuracy of $|\alpha| \leq 10^{-7}$ has been obtained, but no paper appeared. This type of experiment has also been interpreted as the proof of the isotropy of the one–way velocity of light since in these experiments light only runs from the emitter at the center of rotation to the absorber mounted on the rotors. However, during the rotation a constant distance from the axis of rotation to the rotor has been taken as granted. Since this constancy depends on the synchronization procedure, these experiments finally also test only the two–way velocity of light.

Other aspects of violations of SR are an anomalous inertial mass tensor [21] in the Schrödinger equation or orientation–dependent spin effects for massive particles, and effects related to higher order derivatives in the Maxwell or Dirac equations. Anomalous mass tensors are looked for in the famous Hughes-Drever experiments [22, 23] and modern versions of it [24–26] constraining these effects by an order of 10^{-30}. Also anomalous spin effects are absent to the order of 10^{-31} GeV [27]. Spectroscopy of anti–hydrogen which may yield information about the validity of the PCT symmetry is in a planning status.

For more complete recent reviews on the experimental verification of SR the reader may consult [28,29]. In [28] also a comparison of kinetic and dynamical test theories has been presented.

2.1.3 Interpretation

One should note that in experiments testing the isotropy of the speed of light the measured quantity is the difference of two frequencies defined by the boundary conditions of standing electromagnetic waves in resonators $\nu = nc/(2L)$ where L is the length of the resonator and n the mode number. It is obvious that any orientation dependence of the length in the case of a not carefully enough carried through experiment might have the same effect as an orientation dependence of the speed of light. Therefore, any influences on the length like temperature variations, tilts, aging of the material, etc. have to be shielded, eliminated or subtracted. At the end, the isotropy of the speed of light is tantamount to a realization of a length standard with the same accuracy. (The same holds for interferometric experiments of Michelson–Morley type.)

A further point has to be considered: If the velocity of light is anisotropic or depends on the velocity of the laboratory, then the Maxwell equations must have a modified form because light has to be derived from Maxwell equations. Due to the covariant structure of the Maxwell equations, a modification which results in a modified speed of light also will modify the Coulomb's law. This in turn will influence the properties of the resonators and, thus, has to be taken into account for a consistent interpretation of these experiments [30]. Also properties of electrons contribute to a proper interpretation of these experiments [31].

The same holds for the independence of the speed of light from the velocity of the laboratory though the situation is a bit more complicated. In these kinds of experiments the frequency defined by the resonator is compared with atomic or molecular clocks. This means that the universality of the behavior of different clocks in inertial systems in different states of motion is tested. Again, a modification of the properties of the speed of light results in modified Maxwell equations and in a modified Coulomb law which again modifies the properties of the resonator but also the atomic and molecular energy levels which govern the properties of clocks. This again has to be taken into account for a consistent interpretation [32].

2.2 General Relativity

2.2.1 The principles

The Theory of General Relativity (GR) is based on three principles which together form the Einstein Equivalence Principle: (i) the Universality of Free Fall (UFF), (ii) the Universality of the Gravitational Redshift (UGR), and (iii) the local validity of Lorentz invariance. (i) has been tested with an accuracy of $5 \cdot 10^{-13}$, (ii) with an accuracy of $7 \cdot 10^{-5}$ (see the article of C. Will in this issue), and the status of Lorentz invariance has just been described above.

2.2.2 The experimental status

The universality of free fall has been confirmed with high accuracy for neutral bulk matter to a precision of $5 \cdot 10^{-13}$ in the Eötvös factor [33]. For other types of particles or forms of energy the precision is much worse. For charged particles this universality has been tested to the 10% level [34], for spin to the 10^{-8} level [35]. No experiment has been done for antimatter. Using Lunar Laser Ranging, that is, the precise determination of the distance of the Earth–Moon system with an accuracy of 1 cm, this universality has also been confirmed for the gravitational energy with $\eta \leq 1.3 \cdot 10^{-3}$. There are predictions that in string theory induced dilaton scenarios or in quintessence models the UFF might be violated at a level of up to 10^{-13} [36,37].

For various types of clocks (atomic clocks based on electronic transitions and on hyperfine transitions, molecular clocks based on vibrational and rotational transitions, microwave and optical resonators) the universality of their redshift in the gravitational field has been examined with an accuracy of up to $3 \cdot 10^{-5}$ [38]. There are plans to test the Universality of the Gravitational Redshift for anti–clocks by using anti–hydrogen and also to perform free fall tests with anti–hydrogen to test the UFF for antimatter [39].

2.3 Interpretation

The important result of the Einstein Equivalence Principle is the universality of the gravitational coupling. In particular all clocks show the same redshift when moving together in the gravitational field. This yields the particular feature that for the definition of time gravity can completely be neglected as far as the difference of the gravitational potential is small over the extension of the apparatus. Otherwise the gravity gradient has to be taken into account. This has to be considered in future for high precision atomic clocks which may be sensitive to changes in height at the order of 1 cm.

3 The definition of time

Time is the most fundamental category and notion in science. All physical phenomena have to be described in relation to time. Operationally, time can be defined by any monotonical process. The unit of time is given by a periodic phenomenon like days and years and fractions of it or by the motion of a pendulum. The most precise "pendula" today are atomic clocks. The monotonic process is the growth of the phase of a quantum state with time, the proportionality being the frequency or the energy of that state, the periodic phenomenon in this case is the real part of the wave function. The measurement of time then is a comparison of events with such a periodic phenomenon.

Since there are many periodic phenomena available there are many definitions of time. These phenomena are, e.g., the revolution of the Earth around the Sun and the rotation of the Earth, then atomic and molecular spectra based on electronic, hyperfine, vibrational and rotational transitions, and, finally, pulsars and binary systems.

It is important to note that different definitions of time rest on different physical laws. It is a particular feature of our physical world that all these definitions – at least up to the present accuracy of these clocks – lead, up to constant rescalings, to the same definition of time.

Historically, the most obvious periodic phenomenon is the rotation of the Earth. Therefore, the unit of time, the second, was defined to be the 86,400th part of the mean solar day. The exact definition of "mean solar day" was based on astronomical theories. However, the measured irregularities in the rotation of the Earth could not be taken into account by the theory and, thus, cannot be corrected, and have the effect that this definition does not allow to achieve the required accuracy. In order to define the unit of time more precisely, the 11th Conferénce Géneral de Poids et Measures (CGPM) (1960, Resolution 9) adopted a definition given by the International Astronomical Union (IAU) which was based on the tropical year (a tropical year is the mean interval between vernal equinoxes which are defined to be that date (approx. March 21 in the northern hemisphere) when night and day are nearly of the same length and the Sun crosses the celestial equator (i.e., declination 0) moving northward). However, already at that time, experimental work had shown that an atomic standard of time interval, based on a transition between two energy levels of an atom or a molecule, could be realized and reproduced much more precisely. Considering that a very precise definition of the unit of time is indispensable for the International System, the 13th CGPM (1967–1968, Resolution 1) replaced the definition of the second by the following:

> The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom. This definition refers to a cesium atom at rest at a temperature of 0 K.

The last remark was intended to make it clear that the definition of the SI second is based on a Cs atom unperturbed by black–body radiation, that is, in an environment whose temperature is 0 K, and that the frequencies of primary frequency standards should therefore be corrected for the shift due to ambient radiation. Today the best clocks can realize this unit of time with an accuracy of 10^{-15}.

The international atomic time TAI (Temps Atomique International) is defined by comparing and averaging the times provided by more than 200 clocks at more than 50 locations, e.g. at the various bureaus of standards around the world, that is, the PTB (Physikalisch–Technische Bundesanstalt) in Germany, Observatoire de Paris in France, USNO (US Naval Observatory) and NIST (National Institute of Standards and Technology) in the US, etc. just to mention a few, see Fig. 1. The time scales defined by these clocks are called TA(k), where k labels the various laboratories. These clocks run independently of one another and their data are combined to generate a perennial time scale. This scale is more stable and more accurate than that of any individual contributing clock. The scale is based on the results of local clock comparisons in the laboratory, and often has an uncertainty of less than 100 ps. (See e.g. [40] and the web pages of the Bureau International des Poids et Measures (BIPM), http://www.bipm.fr.)

The synchronization and comparison of these clocks operating in laboratories all over the world is a very important task for global time metrology. For that, among other techniques, the satellites of the GPS are used. They are equipped with cesium and rubidium clocks which broadcast time signals and thus provide a satisfactory solution to this problem. Clocks in two distant laboratories are compared individually with a clock on board a satellite which is visible simultaneously from both laboratories and the difference is calculated. For a comparison extending over ten minutes, the uncertainty thus obtained may be a few nanoseconds. To reduce these uncertainties the data must be considered very carefully: The results obtained from views that are not strictly simultaneous must be systematically rejected and a correction must be applied to take into account the exact position of the satellite. Therefore exact data can be obtained only a few days later.

The clock comparison via the GPS satellites is carried out using by TWSTFT (Two–Way Satellite Time and Frequency Transfer). The application of GPS common–view C/A-code and TWSTFT to clock comparisons, as well as the development of more stable commercial cesium standards, have improved the accuracy of timing data by nearly two orders of magnitude over the last two decades. Another improvement is the increasing number of multichannel GPS receivers available in time laboratories. All GPS links in TAI are corrected using ionospheric maps and precise operational satellite ephemerides produced by the International GPS Service (IGS). Changes in ionospheric parameters, the calibration of GPS, GLONASS and TWSTFT equipment, use of post-processed GPS and GLONASS precise ephemerides, improved knowledge of the coordinates of national laboratories, and correlations between clocks have to be included properly.

Other methods under study for clock comparison are bidirectional techniques based on the transmission of an optical or radio frequency signal from one laboratory to another and back, via a satellite. Such methods should lead to sub-nanosecond accuracy. All these methods of time comparison are subject to relativistic effects which may exceed 100 ns, so corrections must be applied to take them into account.

The optimal combination of all the results of comparisons between the clocks in the national laboratories results in the International Atomic Time (TAI), approved by the 14th CGPM in 1971 (Resolution 1)

> International Atomic Time (TAI) is the time reference coordinate established by the Bureau International de l'Heure on the basis of the readings of atomic clocks operating in various establishments in accordance with the definition of the second, the unit of time of the International System of Units.

In the framework of GR, TAI must be regarded as a time coordinate. In order to compare these clocks in a consistent way one has to include the fact that these clocks are in different states of motion and on

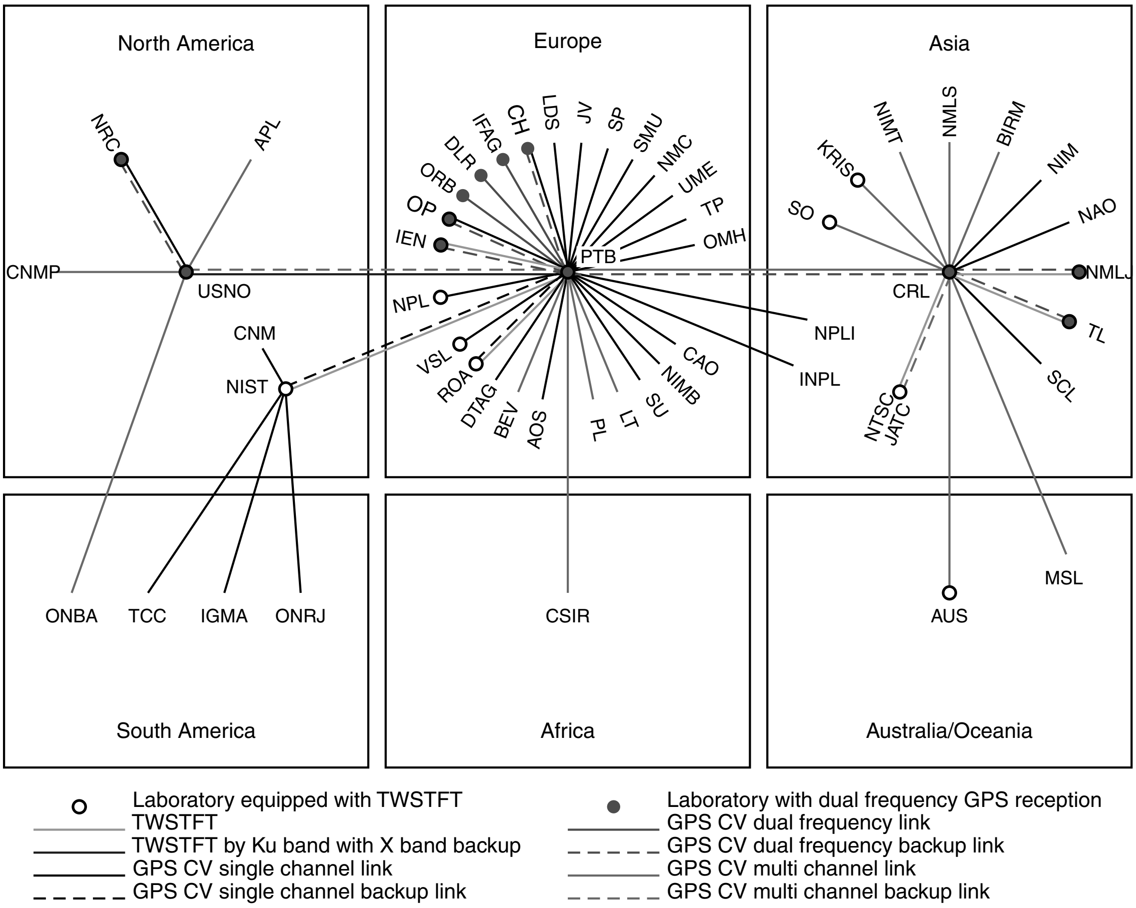

Fig. 1 (online colour at: www.ann-phys.org) The organization of the international time links: comparison and synchronization between the various clocks on Earth provide the TAI. Each continent has a star–like structure with a central laboratory. The acronyms are abbreviations for the various laboratories (e.g., PTB = Physikalisch Technische Bundesanstalt (Braunschweig), USNO = US Naval Observatory, OP = Observatoire de Paris, DLR = Deutsches Zentrum für Luft– und Raumfahrt (Oberpfaffenhofen), DTAG = Deutsche Telekom AG (Darmstadt), IFAG = Bundesamt für Kartographie und Geodäsie (Fundamentalstation Wettzell), CRL = Communications Research Laboratory (Tokyo), AUS = Consortium of laboratories in Australia, CSIR = Council for Scientific and Industrial Research (South Africa), etc.).

a different gravitational potential. All clocks refer to the geoid, which is the surface of the constant total potential consisting of gravitational and centrifugal potential, see below. The definition of TAI was therefore completed as follows:

> TAI is a coordinate time scale defined in a geocentric reference frame with the SI second as realized on the rotating geoid as the scale unit.

TAI is a uniform and very stable time scale. However, daily life in long terms is related to the rotation of the Earth which is irregular and on large scale slows down. For public and practical reasons one needs a time scale which is as precise as TAI but is related to the rotation of the Earth. Such a timescale is the Coordinated Universal Time UTC which is obtained from TAI by introducing from time to time a leap second (the next leap second will be inserted at the end of December 31, 2005, at UTC midnight). This is

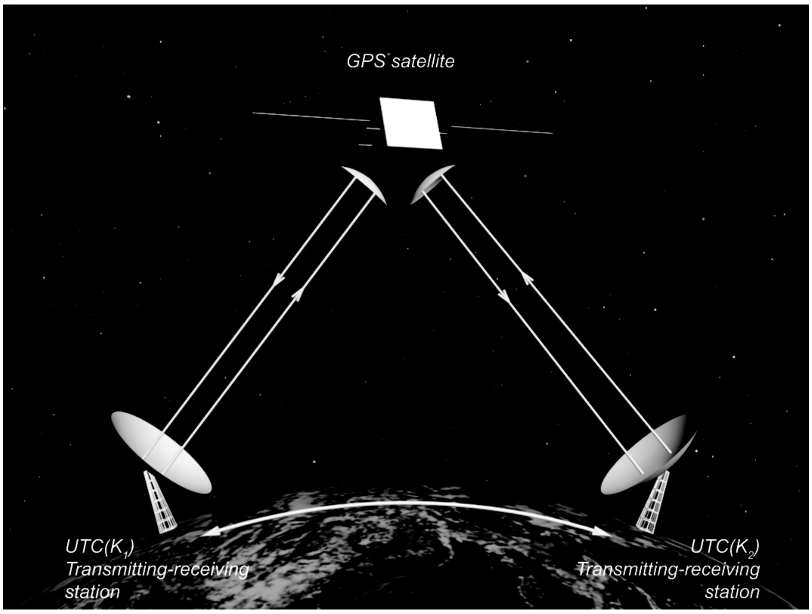

Fig. 2 The definition of the TAI required a time transfer between clocks on Earth and GPS satellites. For that special and general relativistic corrections are needed. Future generation clocks will require second order correction.

organized by the International Earth Rotation Service IERS. See Fig. 3 for a small overview over existing time scales and their relation.

Relativity, Special and General, acts in the same way on all clocks. This is a matter of consistency in the sense that otherwise different clocks and clocks at different locations and in different states of motion will all show different drifts. If the drifts are quadratic what in general is not the case, one may modify the formalism to characterise stability, see e.g. the comparison of Earth bound clocks with pulsar clocks which necessarily show drifts [41].

The clocks on Earth are located at different heights and altitudes on Earth, and thus, experience different gravitational potential and move with different velocities. The same holds for the clocks on satellites. This has to be taken into account for the comparison of all these clocks. The comparison is carried through using the proper time τ of the real clocks compared with the time of a fiducial clock with time t located at the Earth's geoid

$$\Delta\tau = \left(1 - \frac{U(x_1) - U_0}{c^2} - \frac{1}{2}\frac{v^2}{t^2} + \boldsymbol{\omega} \cdot \frac{d\boldsymbol{A}}{dt}\right) dt\,. \tag{1}$$

Here $U(x)$ is the gravitational potential at position x including all multipole contributions of the Earth as well as the centrifugal potential and U_0 is the potential of the geoid. Furthermore, v is the velocity of the clock with respect to the Earth, $\boldsymbol{\omega}$ the angular velocity of the Earth and $\boldsymbol{A}$ is the area a vector sweeps out from the center–of–mass of the Earth to the clocks [40,42]. This formula is built into the programs used in the GPS receivers, see [43] for a review. The importance is becoming clear by noting the numbers: While clocks today have a relative precision of the order 10^{-15}, the above contributions to GPS or Galileo satellites ($h \approx 20200$ km) yield the following relative effects: gravitational potential $(U(x) - U_0)/c^2 \approx gh/c^2 \sim 2 \cdot 10^{-9}$, gravitational quadrupole $\sim 10^{-13}$, centrifugal potential $\frac{1}{2}\omega^2 h^2/c^2 \sim 2 \cdot 10^{-12}$, velocity $\frac{1}{2}v^2/c^2 \sim 10^{-11}$. For an orbit of a clock around the Earth, the Sagnac effect gives a time difference of $\sim 10^{-7}$. All these effects are much bigger than the present accuracy of clocks and, thus, have to be taken into account.

For future clock generations like PHARAO with a projected accuracy of 10^{-16} these effects are even more pronounced. Furthermore, since optical clocks are expected to reach an accuracy of 10^{-18} even second order effects which are of the order $5 \cdot 10^{-18}$ will have to be taken into account [44]. Also terms of the order $1/c^3$ related to the quadrupole moment of the Earth which turn out to be of the order 10^{-16} certainly

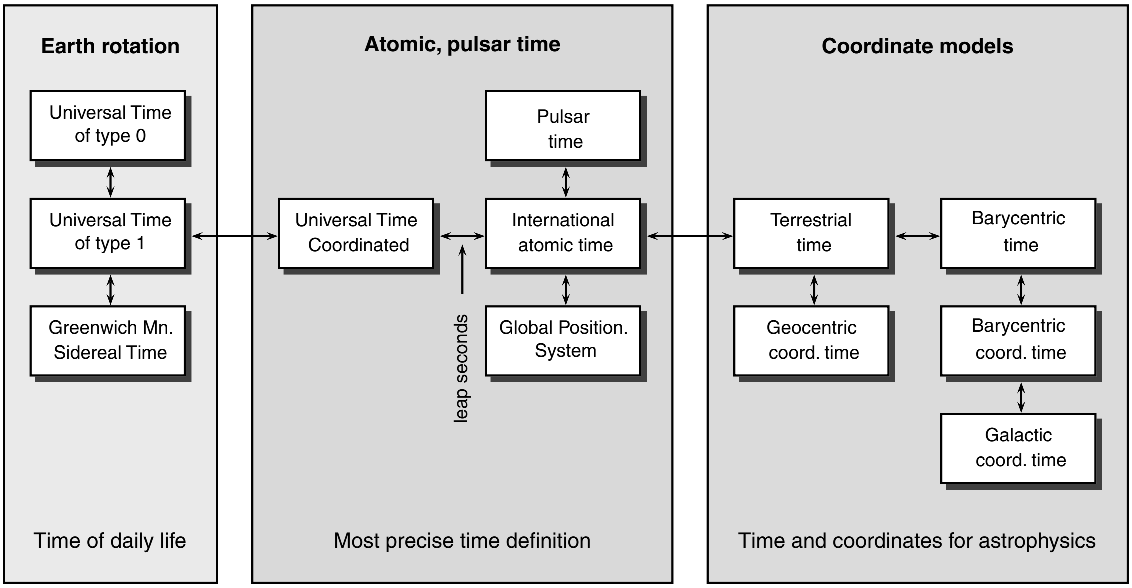

Fig. 3 (online colour at: www.ann-phys.org) Time scales and relations between them. For all relations between the various time scales but the astrophysical ones, one needs SR and GR. See [42,45]. In order to account for the variability of the Earth rotation with respect to the atomic time, sometimes additional leap seconds have to be inserted.

have to be considered in the future [44]. Clocks with an accuracy of 10^{-18} can "see" the Doppler shift of the continental drift (which is of the order of 1 cm/y), or they "see" the difference in height of 1 cm. In many places on Earth, the height is varying irregularly (that is, the variation cannot be modelled precisely enough) by much more than 1cm. Therefore it is not very convenient to use a clock with a height which is less precisely defined than the accuracy of the clock as a time standard. As a consequence, in future very precise clocks and, thus, the time standard on Earth have to be operated in space.

4 The definition of length

The first international valid definition of the metre has been set up by the first CGPM in 1889. It is based upon the international prototype of platinum-iridium. The original international prototype of the metre is still kept at the BIPM in Paris under the conditions specified in 1889.

Due to the inaccuracy of taking duplicates and of comparing various lengths this definition was replaced by the 11th CGPM in 1960 using a definition based upon a wavelength of krypton 86 radiation. This definition was adopted in order to improve the accuracy with which the metre may be realized.

Further technical improvements in the realization of the unit of time made the second even more precise than – after transforming the time unit into a length unit using the velocity of light – the metre. Therefore, the definition based on a particular wavelength was replaced in 1983 by the 17th CGPM (Resolution 1):

> The metre is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.

Note that the effect of this definition is to fix the speed of light at exactly 299 792 458 m/s.

The importance of the constancy of the speed of light is very clear: If the speed of light is not constant, then the metre may depend on the orientation or on the state of motion of the laboratory. More precisely, one may obtain a different metre if one prepares a metre according to the above definition in one direction and rotates this metre into another direction and compares it with a metre prepared in that direction. And similarly, if a metre has been prepared in one reference system and brought to another system moving with respect to the first one, then this metre may be different from that prepared in the moving system. Though one may handle this situation in principle, the definition of the metre then will be much more complicated since one has to take the state of motion into account which may be difficult to determine.

Since the modern definitions of the resistance (ohm) and the voltage (volt) which are based on the quantum Hall effect and the Josephson effect depend on its description using the Schrödinger and the Maxwell equations, and since both of these equations rely on the validity of Special and General Relativity, also these definitions of the ohm and the volt depend on the validity of the Einstein Equivalence Principle. Compare in this connection the proof that within the Einstein Equivalence Principle the quantum Hall effect is not influenced by the gravitational field [46]. Also a new definition of the kg via the Watt balance [47] which is based on the quantum Hall and Josephson effect as well as on the UFF rely on the validity of SR and GR.

5 GPS

The global positioning system GPS consists of more than 24 satellites at 20,200 km height. Each is equipped with two cesium/rubidium atomic clocks or hydrogen maser clocks, see [43] for a recent review.

The primary objective of the GPS is positioning on the Earth and in nearby space. Meanwhile this is important for new commercial markets like the collection of highway tolls in Germany with a volume of many billion Euros. Today, we have an accuracy of a few m which in future may come down to the sub–mm level. By using better clocks and better devices like multi–frequency links which can reduce the influence of the Earth's atmosphere on the signal propagation, the accuracy may still be improved. Of course, nobody needs to know his position to that precision. However, it might be of big advantage if one is able to monitor the motion of the ground in certain regions on Earth to that precision in order to detect variations which may give some new insight into the dynamics of the Earth and also may help in the prediction of earthquakes.

In order to reach such a precision, better clocks are needed what in turn requires the analysis of the received signals to second order in the relativistic approximation. This is a theoretically highly non–trivial task and requires experts in that field. It also needs to set up better Earth models which also requires higher order relativistic approximations.

It is remarkable that the GPS also has been used directly for pure science, namely for a new test of SR [48].

6 Atomic and molecular physics

The well–known and very basic spin–orbit coupling is of relativistic origin; the relativistic Thomas precession of the spin of the electron while being accelerated in the electric field of the nucleus essentially contributes to that effect. Therefore already at a very basic level the structure of the atoms is determined by relativity. Also the energy levels not related to spin are modified by relativistic corrections. Classically speaking, these corrections can be related to the relativistic mass increase. As a famous example, the ionization energy of gold is heavily determined by these relativistic corrections. This comes from an additional contraction of the 6s orbital which makes the atom more stable. This relativistically based stability also influences the property of materials containing gold, a fact which even obtained its own name, namely the *gold anomaly*. Therefore, relativity is essential for the material sciences. For a survey of these effects, see [49]. A further effect on the energy levels is the Lamb shift, the interaction of the electronic levels with the vacuum fluctuations of the electromagnetic field.

From that it is clear that also a variety of more subtle properties of atoms and molecules depend on relativity. Indeed, it has been proven not only qualitatively but also quantitatively that relativistic effects play an essential role in the molecular electronic structures of heavy–element systems and of electron correlation effects.

7 Earth sciences

One important information about the Earth is the precise knowledge of its gravitational field which, e.g., gives information about the Earth's mass distribution. This has been modeled up to the 100th multipole moment and is still measured with the satellites LAGEOS, CHAMP, and GRACE[1], and can be extended to the 360th moment by including terrestrial data, see also [50]. The gravitational field gives information about the constituents of the Earth's mantle which is important for the search of natural resources. The task today and in the future is not only to measure the non–spherical shape of the Earth as precise as possible but also to determine the time–dependent phenomena like atmospheric dynamics, the ice shell at the polar caps, the contionental water resources, continental drifts, the dynamics of the oceans, the flows in the inner core of the Earth etc., which are all much more important to daily life. In future, small variations in the Earths gravitational field on ground may also be determinable by the use of the new generation of optical clocks.

One tool for doing so is the precise measurement of the rotation rate of the Earth which uses Very Long Baseline Interferometry (VLBI), and for the functioning of that relativity is needed. VLBI compares the difference in the time–of–arrival of a signal from very distant objects like pulsars from which the direction of the pulsar with respect to the two ground stations can be determined very precisely. For that, a very good synchronization (which includes relativistic effects) between the clocks at the various VLBI stations is needed. Since the position of the pulsars do not change, any anomalous change in the determined direction is related to an anomalous rotation or change of the position of the ground stations. The precisions of VLBI as well as that of Satellite Laser Ranging (SLR), GPS, and of DORIS is so high that it is possible to see climate influences in the rate of rotation, the El Niño effect, or the warming up of regions of the oceans, for example, see Fig. 4.

One aspect of the rotation is the axis of rotation, the other the period. The latter is described by the length of the day (LOD). The variation of the LOD has various reasons, most of them are easily related to well known phenomena. The variations which occur with a period of one or half a day are related to ocean tides, variations with a bi-weekly period can be related to the deformation of the Earth by the gravitational field of the Moon, etc. Also the El Niño effect can be identified in the LOD data, see Figs.5 and 6. The El Niño effect and other atmospheric phenomena are easily related to LOD data. However, a warming up of the ocean which is not so easily detected directly, in particular, when it occurs in deeper regions may be a

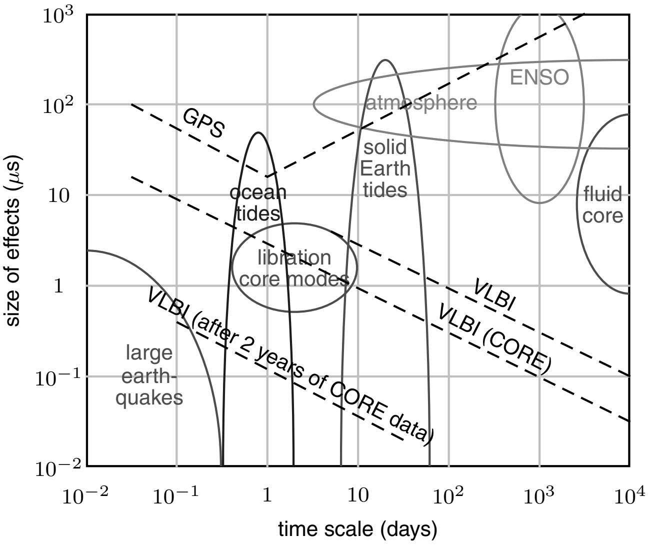

Fig. 4 (online colour at: www.ann-phys.org) The influence of various Earth dynamical effects on the rotation of the Earth and the corresponding accuracy of various devices. CORE = Continuous Observation of the Rotation of the Earth. ENSO = El Niño Southern Oscillation.

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[1] The ESA mission GOCE to be launched in 2006 should be able to determine the multipole moments up to the 200th order.

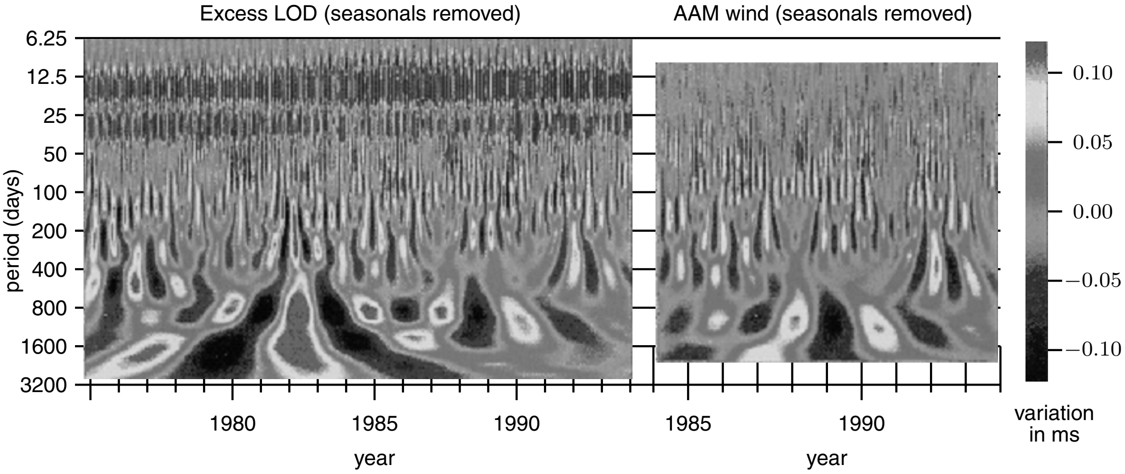

Fig. 5 (online colour at: www.ann-phys.org) The fast Fourier transform of the variation of the Earth rotation. From the variation of the LOD one can obtain information about the atmospheric state of the Earth. AAM = Atmospheric Angular Momentum.

candidate for future analysis of LOD data. Generally, there is a lot of information encoded in the rotation of the Earth. In order to handle the increasing accuracy of the measurements complicated models of the Earth's rotation on a post–Newtonian level have to be designed [51,52].

The very precise identification of the rotation of the Earth is also of big importance for astrometry. All the observations are made from observatories on the surface of the Earth which constitutes an accelerated reference frame with a very complicated rotation. One has to know this rotation in order to be able to represent the astrophyical events in an astrophysical frame like the barycentric or the galactic or the frame.

Also the positions on Earth vary. In particular the positions of laboratories and observatories vary. This is important for the synchronization, and also for the determination of the position of astrophysical events which coordinates in some astrophysical coordinate system like the barycentric one has to be calculated from the observations which are made in the rotating coordinates on Earth defined by all the observatories. For example, the daily and yearly variation of the positions of the Pioneer 10 and 11 spacecrafts as calculated from the observations from the tracking stations on Earth have been assigned to an uncomplete modeling of the transformation from Earth coordinates to astrophysical coordinates [53].

8 Summary

Today everyday technical applications of Relativity, Special and General, can be found at many instances. Relativity will become even more important in the not too far future with, e.g., applications to Earth sciences. Therefore, SR and GR obtained the status of a service discipline for daily life applications. In future high precision time definition, Earth sciences etc., higher order special and general relativistic effects routinely have to be taken into account. This implementation is a highly nontrivial combination of experiment/observation, modeling and implementation of relativity and, thus, requires the full skills of Special and General Relativity.

A further interesting aspect is that the new development of high precision technologies in most cases goes hand in hand with fundamental physics tests. This can be seen, e.g., from the famous Hafele–Keating experiment: Shortly after the construction of a new atomic clock with – at that time – highest accuracy which has been motivated by technological requirements only, Hafele and Keating recognized that these devices can also be used to measure the time dilation with the macroscopic transport of clocks for the first time [55,56]. Another example is neutron and atom interferometry. After the development of neutron interferometry this

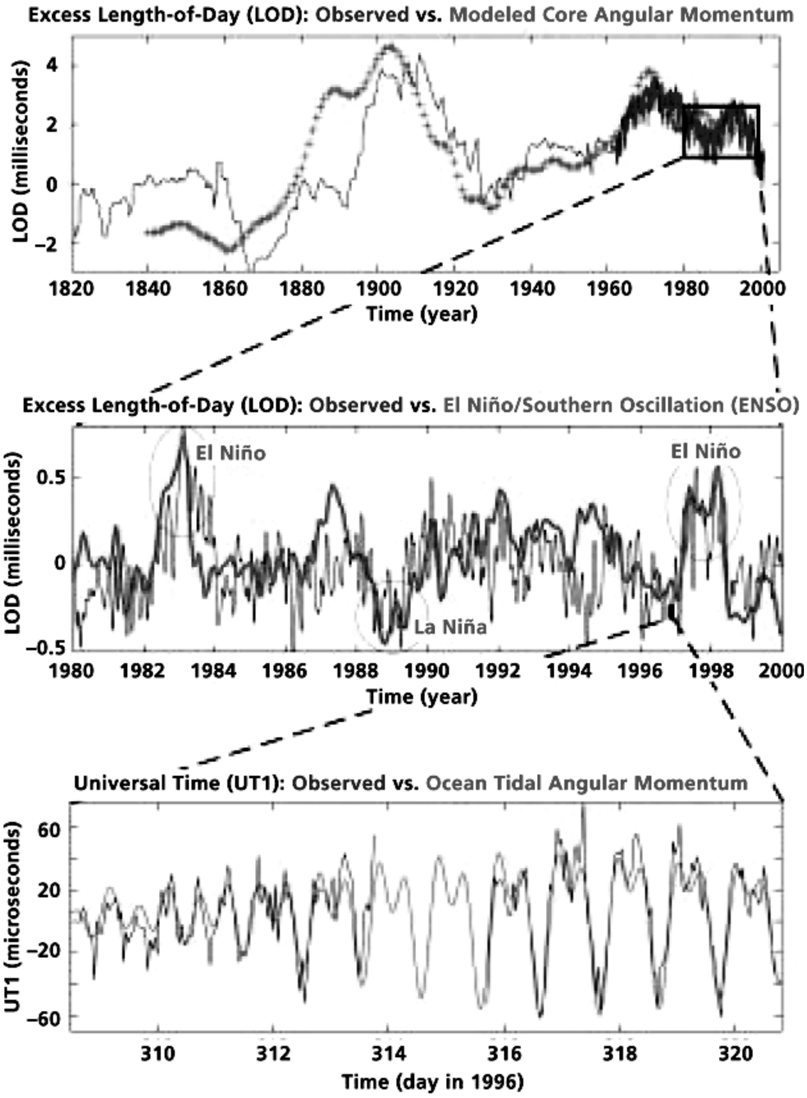

Fig. 6 (online colour at: www.ann-phys.org) Variations of the length of the day and the identification of the El Niño effect (from [54]).

device has not only been used for material sciences, it has almost immediately been taken for a first direct demonstration of the influence of the gravitational field on a quantum wave function. Also, further aspects of quantum theory, like its linearity, the 4π–periodicity of spinors, decoherence, delayed–choice, Aharonov–Casher phase shifts, have been explored. The same applies to atomic interferometry. Atomic interferometry is a by–product of the laser–cooling technique and has been very quickly used for tests of the Equivalence Principle and high precision measurements of fundamental constants like the fine–structure constant though the primarily intended use was its application in geology and also in the military. As a third example we may mention the Gravity Probe B mission. The original question was a military motivated wish of high precision navigation with ultra precise gyroscopes [57]. For that one has to take into account the gravitomagnetic precession of gyroscopes. Since this effect is really small and has no influence of ordinary navigation, its detection then results in a pure fundamental physics task [58]. However, the realization of this mission induced also the development of new attitude control techniques which are not only important for further fundamental physics space mission but also for missions measuring the gravitational field of the Eaerth. As a conclusion one may state that each development, either in technology or in fundamental physics, will have impact in both areas, technology and fundamental physics.

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