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The search for quantum gravity effects I

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ABSTRACT In this contribution the search for effects from possible theories of quantum gravity is reviewed. In order to distinguish quantum gravity effects from standard effects, first the standard theory and the principles it is based on has to be described. We show that standard physics (the Maxwell equations, the Dirac equation, gravity as a metric theory) is completely based on the Einstein equivalence principle, EEP (for obtaining the Einstein equations, some more requirements are needed). As a consequence, all deviations from the EEP are related to new effects originating from quantum gravity. The variety and structure of these effects is described and the expected magnitude of the effects and a corresponding strategy for the search for these effects are discussed. We stress the advantages of space for performing experiments searching for quantum gravity effects. At the end we make some remarks concerning the daily-life applications of high-precision techniques.

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1 Introduction

The questions which we would like to discuss in this contribution are

- Why do we need a theory of quantum gravity?
- What are the possible effects of quantum gravity?
- Where can we expect effects from quantum gravity?
- How big can these effects be?
- What is the best strategy to search for quantum gravity effects?
- What kinds of tests do we have to pursue in the search for quantum gravity?

All these questions are related to the issue of quantum gravity phenomenology.

Further questions, more specifically related to the present workshop "Quantum Mechanics for Space", are

- How is quantum theory related to quantum gravity?
- Has one to go to space in order to improve the experimental capabilities?

Before discussing these questions, we should outline the present status of physics in order to understand what kinds of effects we can expect from a quantum gravity. We would also like to refer to other survey articles on that topic by Sarkar [1], Amelino-Camelia et al. [2–4], and Mattingly and co-workers [5, 6].

2 Standard physics and its underlying principles

In order to get into the subject and the important questions, we describe the present structure of standard physics, which is deeply linked to the structure of general relativity and which itself is given by the structure of the 'rest' of physics, that is by the structure of the equations of motion of the matter fields (which include the non-gravitational interactions like electromagnetism and the weak and strong interactions).

The structure of general relativity and the equations of motion of the non-gravitational fields, the matter fields, are mainly determined by the Einstein equivalence principle (EEP); see the scheme Fig. 1. This scheme has two aspects:

1. The EEP implies that gravity has to be described by a pseudo-Riemannian space–time metric.
2. The EEP also fixes the structure of the equations of motion for the matter fields and interactions, that is, the Dirac equation, the Maxwell equations, etc.

First we discuss this principle in general and then draw specific conclusions for the structure of the equations of motion for the matter fields and, as a consequence, for the structure of the gravitational field.

2.1 *The Einstein equivalence principle*

The structure of general relativity is mainly determined by the EEP (see Fig. 1), which includes

1. The universality of free fall (UFF): this principle states that in the gravitational field all kinds of structureless massive test particles[1] fall in the same way. In order to test that one has, in principle, to compare the free fall of all sorts of available materials, which means that one has to perform a huge number of experiments. However, in the frame of elementary particle theory this just means that all elementary particles fall in the gravitational field in the same way. Therefore, one has to perform only as many experiments

[1] Test particles are particles which are sufficiently small so that their own gravitational field can be neglected.

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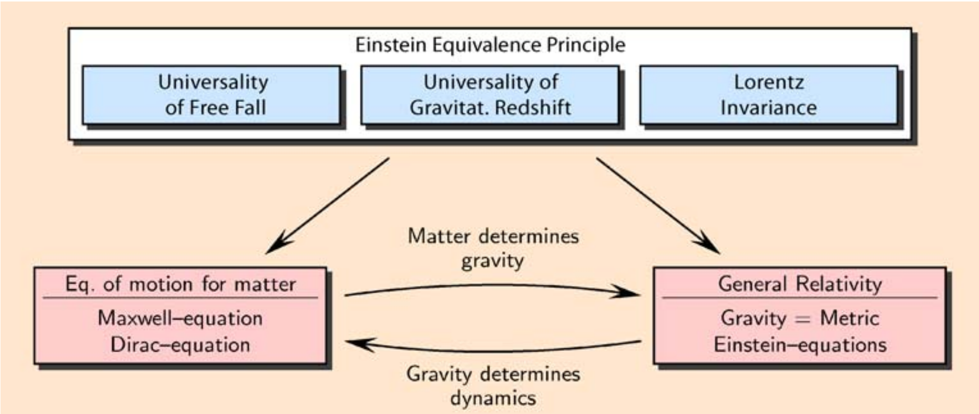

FIGURE 1 The scheme of general relativity: the EEP determines the structure of the gravitational field as well as of the equations of motion

as there are fundamental elementary particles. In practice, one carries through experiments with various macroscopic species of materials and then analyses the results in terms of the coupling of the elementary particles to the gravitational field. For the theoretical analysis, one introduces for each fundamental elementary particle a different coupling to the gravitational field. The analysis of the experimental results then gives estimates of the differences of the coupling parameters. If UFF is exactly fulfilled then all the coupling parameters should be the same. This equality of all these parameters is underlying Einstein's general theory of relativity. In principle this is an amazing fact: why do all particles behave in the same way? Wherefrom do all the particles know how the other particles behave? This fact is characteristic for the gravitational interaction. No other interaction shares this property.

2. The universality of the gravitational red shift (UGR): this principle states that all kinds of clocks based on non-gravitational physics (pendula or sand clocks are not allowed) behave in the same way when transported together through a position-dependent gravitational field. This again means that all particles and all (non-gravitational) interactions (also represented by particles) couple in the same way to the gravitational field. Again, one has to test this for all kinds of clocks and analyse the result in terms of the coupling of elementary particles to gravity.
3. The local validity of Lorentz invariance (LLI): the third underlying principle is the local validity of Lorentz invariance. This means that the outcomes of all local small-scale experiments are independent of the orientation and the state of motion of the laboratory. This means that by means of experiments it is not possible to single out a particular reference system. In particular, this means that the velocity of light is constant and all limiting velocities of elementary particles are again given by the velocity of light. This again is an amazing fact: wherefrom do all the particles have knowledge of the properties of the other particles? Since LLI is a property which applies to all physics, one has to perform tests with all physical systems. The Michelson–Morley, Kennedy–Thorndike, and Ives–Stilwell tests are the most well known tests of this type exploring the properties of photons, which have to be completed with tests with electrons, protons, etc., which are given by, for example, the Hughes–Drever experiments.

One can show [7] that from these three principles the gravitational field has to be described by a space–time metric (right arrow in Fig. 1). This is not yet Einstein's general relativity. Some more assumptions are needed such as, for example, the non-occurrence of the Nordtvedt effect.

2.2 *Implications for the equations of motion*

The EEP not only determines the metrical structure of gravity but also fixes the structure of the equations of motion of the matter content in the universe, that is, of the

- the Maxwell equations,
- the Dirac equation (which in the non-relativistic limit leads to the Schrödinger equation),
- the structure of the standard model.

Since gravity is what can be explored by the dynamics of test matter like material point particles, light rays, matter fields, etc., it is clear that any restriction in that dynamics also restricts the degrees of freedom of the gravitational field.

2.2.1 Implication for point particles and light rays. As a particular well worked out example of that approach, one has to mention the Ehlers–Pirani–Schild (EPS) approach to the gravitational field [8, 9]. This approach is based on the most simple physical objects one can think of: structureless point particles and light rays. Assuming (i) that there are only two light rays connecting one space–time point with a (nearby) trajectory introduces a conformal metrical structure, that is, a Riemannian metric up to a position-dependent conformal factor. This establishes a point-wise Lorentzian structure. Assuming furthermore (ii) that the trajectory of a structureless uncharged point particle is completely determined by stating its position and velocity – what is equivalent to the UFF – gives a path structure (the set of all paths up to a reparametrization) on the manifold. Requiring (iii) that the path structure is compatible with the conformal structure in the sense that for each direction inside the light cone there is a particle leads to a Weylian structure (otherwise it would be possible to single out a preferred frame, which violates LLI). The last requirement, namely that there is no second clock effect which amounts to requiring the universality of clocks in gravitational fields and, thus, the validity of the UGR, finally gives a Riemannian structure, that is

$$v^\nu D_\nu v^\mu = \alpha v^\mu \tag{1}$$

as the equation of motion for a point particle, where v^μ is the 4-velocity of the point particle and α is an arbitrary function. Here

$$D_\nu v^\mu = D_\nu v^\mu = \partial_\nu v^\mu + \left\{ {}^{\mu}_{\nu\varrho} \right\} v^\varrho \tag{2}$$

is the covariant derivative, where

$$\left\{ {}^{\mu}_{\nu\varrho} \right\} = \frac{1}{2} g^{\mu\sigma} \left(\partial_\nu g_{\sigma\varrho} + \partial_\varrho g_{\sigma\nu} - \partial_\sigma g_{\nu\varrho} \right) \tag{3}$$

is the Christoffel symbol which is calculated from the Riemannian metric $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$ defined through $g^{\mu\varrho} g_{\varrho\nu} = \delta^\mu_\nu$.

2.2.2 Implication for spin-$\frac{1}{2}$ particles. Another example of a physical system are spin-$\frac{1}{2}$ particles described by a multicomponent complex vector field. Though the operational definition of such a field has not yet been worked out in a satisfactory way, the consequences of the requirement of the EEP on the dynamics of such a matter field are quite clear.

The first step is to set up a general dynamics for such a matter field by requiring the existence of a mapping $\psi_t = U(t, t_0)\psi_{t_0}$, where t is related to a foliation of space–time. Now we require this dynamics to be linear, which implies that U is a linear operator. The requirement that there are no solutions propagating with an infinite velocity has serious consequences: first, it makes the dynamics a local one, that is, the general dynamical equation is equivalent to a linear system of partial differential equations. Furthermore, this requirement implies that the system is of first order [10].

Now, LLI requires that there is only one characteristic cone for this system. This defines a Riemannian metric. Furthermore, the requirement of LLI also implies that there are no other tensor fields which can couple to the matter field. Therefore, we end up with the Dirac equation coupled to a Riemannian space–time metric as the only gravitational field [10]:

$$0 = \mathrm{i}\gamma^\mu D_\mu \psi - m\psi , \tag{4}$$

where ψ is a 4-spinor and γ^μ are the Dirac matrices fulfilling the Clifford algebra

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \tag{5}$$

and

$$D_\mu \psi = \partial_\mu \psi + \Gamma_\mu \psi , \tag{6}$$

with

$$\Gamma_\mu = -\frac{1}{2} \left(D_\mu h^\nu_a \right) h^\varrho_b g_{\nu\varrho} G^{ab} . \tag{7}$$

The $G^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$ are the spinorial generators of the Lorentz transformations, and h^μ_a are the tetrads defined by $h^\mu_a h^\nu_b g_{\mu\nu} = \eta_{ab}$, where η_{ab} is the Minkowski metric. The requirement of UFF or, equivalently, UGR implies that the mass m has to be a constant [10].

2.2.3 Implications for the Maxwell field. A similar procedure can be carried through for the electromagnetic field. Starting from some general Maxwell equations which are linear in the electromagnetic field strength tensor and first order in the derivative, the requirement of LLI again amounts to the requirement that the characteristic cones are not allowed to split. From that consideration a coupling to a space–time metric follows [11, 12]. LLI also requires that there are no other fields the Maxwell equations can couple to. Therefore, we arrive at the ordinary Maxwell equations minimally coupled to the space–time metric

$$D_\nu \left(g^{\mu\varrho} g^{\nu\sigma} F_{\varrho\sigma} \right) = 4\pi j^\mu , \qquad \partial_{[\mu} F_{\nu\varrho]} = 0 , \tag{8}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength expressed in terms of the Maxwell 4-potential A_μ. One interesting feature is that LLI also is responsible for the validity of charge conservation. Also, a coupling to a pseudoscalar field which gives Ni's axion [13] is forbidden, since the derivative of a scalar field is a vector that defines a preferred direction and, thus, breaks Lorentz invariance. Therefore, for the electromagnetic field, LLI alone is enough to show that only a space–time metric can couple to the electromagnetic field.

Another aspect has been discussed by Ni [13], who showed that when using generalized Maxwell equations in order to describe the behavior of a neutral electromagnetically bound system made up of charged particles, this neutral system will violate the UFF. Therefore, the requirement of the UFF also forces the Maxwell equations to be of a certain structure.

2.2.4 Summary. For the cases discussed above, we found that the EEP implies the ordinary equations of motion for the physical system under consideration, which is shown in Fig. 2.

In each case, the gravitational field which is compatible with the EEP is a Riemannian metric. Until now it is allowed that for each physical system we have another metric. However, in a last step in applying LLI one requires that the maximum velocities of all kinds of matter are the same. This implies that the metrics governing the motion of point particles, of the spin-$\frac{1}{2}$ particles, and of the electromagnetic field are all the same. Therefore, we have a universality of the causal cones. As a overall consequence we have that the EEP implies that gravity is a unique metrical theory.

2.3 *Implications of the EEP for the gravitational field*

The second aspect of the EEP is clearly in accordance with the first because the gravitational field and, thus, its structure can be explored by the observation of the dynamics of test matter in the gravitational field only. And, only if the equations of motion of these matter fields possess a certain structure are they are compatible with the metrical structure of the gravitational field.

Now we have to set up the equations from which the space–time metric can be determined. Until now there is no unique way to derive the Einstein field equations from simple requirements. The presently used scheme relates all possible physical sources of the gravitational field, that is, mass density, pressure, mass currents, etc., to various components of

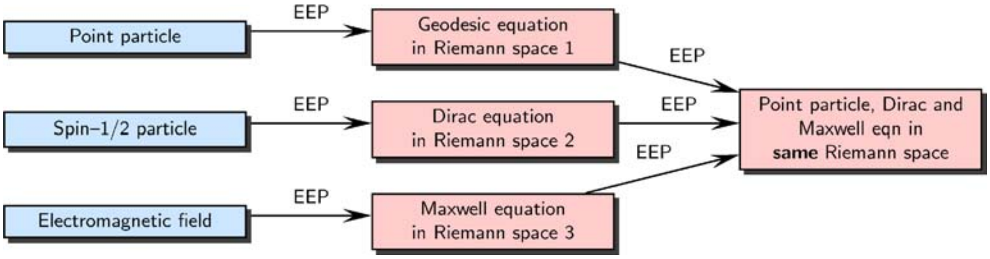

FIGURE 2 Implications of the EEP

the metric in a combinatorial way which is motivated by the determination of the Newtonian gravitational potential from the mass density. This is the so-called PPN formalism. This is a very general parametrization of all metrical gravitational theories. Within this parametrization one can calculate all the well-known measurable effects like perihelion shift, red shift, light bending, etc., and also effects which are not present in Einstein's theory like the Nordtvedt effect, effects with a preferred frame, effects related to momentum non-conservation, etc. A comparison with the conservation of all these effects then shows that the estimates for all these parameters are compatible with the set of parameters characterizing Einstein's general theory of relativity. The compatibility in general is at the 10^{-4} level. As a consequence, the space–time metric is determined from the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} , \tag{9}$$

where

$$R_{\mu\nu} = R^{\varrho}{}_{\mu\varrho\nu} , \qquad R = g^{\mu\nu} R_{\mu\nu} \tag{10}$$

are the Ricci tensor and Ricci scalar and

$$R^{\varrho}{}_{\sigma\mu\nu} = \partial_\mu \left\{ {\varrho \atop \nu\sigma} \right\} - \partial_\nu \left\{ {\varrho \atop \mu\sigma} \right\} + \left\{ {\varrho \atop \mu\tau} \right\} \left\{ {\tau \atop \nu\sigma} \right\} - \left\{ {\varrho \atop \nu\tau} \right\} \left\{ {\tau \atop \mu\sigma} \right\} \tag{11}$$

is the curvature tensor derived from the Christoffel symbol. $T_{\mu\nu}$ is the energy momentum of the matter in the universe and the factor $8\pi G/c^2$ has been determined from the comparison with the Newtonian gravitational field.

From the Einstein field equations we obtain $D_\nu T^{\mu\nu} = 0$, which are the equations of motion of the matter that creates the gravitational field. This gives back the equations of motion for the point particles, the electromagnetic field, and the Dirac equation.

2.4 *The consequences*

All aspects of Einstein's general relativity are experimentally well tested and confirmed. The tests split into tests of the EEP which we have described below, and tests of the predictions of general relativity. No single test contradicts its foundation or its predictions. These predictions are

1. Solar system effects.
   - Perihelion shift: this effect had been observed long before the rise of general relativity. Due to a competing cause by the Sun's quadrupole moment, the relativistic effect can be confirmed at the 10^{-3} level only.
   - Gravitational red shift: the speeding up of clocks when being brought to large heights has been best tested by the GP-A mission, where a H-maser in a rocket has been compared with a H-maser on ground. Any deviation from the Einstein prediction has to be smaller than 7×10^{-5} [14].
   - Deflection of light: this was the first prediction of Einstein's general relativity which was confirmed by observation only four years after setting up the complete theory. Today's observations use Very Long Baseline Interferometry VLBI, which leads to a confirmation of Einstein's theory at the 10^{-4} level [15]. This will be improved by the Gaia mission by several orders of magnitude; see e.g. [16].
   - Gravitational time delay: electromagnetic signals move slower in stronger gravitational fields. Therefore, light or radio signals need a longer time of propagation when the Sun comes nearer to the trajectory of the electromagnetic signals. The best test of this phenomenon has been carried through recently by the Cassini mission, which led to a confirmation of Einstein's theory at the 10^{-5} level [17].
   - Lense–Thirring effect: the rotation of a gravitating body results in a genuine post-Newtonian gravitomagnetic field, which, of course, influences the equation of motion of bodies and also the rates of clocks. The influence of this field on the trajectory of satellites results in a motion of the nodes, which has been measured by observing the LAGEOS satellites via laser ranging. Together with new data of the Earth's gravitational field obtained from the CHAMP and GRACE satellites, the confirmation recently reached the 10% level [18].
   - Schiff effect: the gravitational field of a rotating gravitating body also influences the rotation of gyroscopes. This effect is right now under exploration by the GP-B mission. The science mode and an additional two-month period of post mission calibration and analysis has been completed together. The results of the ongoing data analysis will be presented in spring 2007. It hast been mentioned at the recent MG11 meeting that all preliminary results seem to be in good agreement with the Einstein prediction. The accuracy of the measurement of the Schiff effect should be better than 1%.
2. General relativity in the strong-field regime. This has been proven to be valid to very high accuracy by means of the observation of binary systems.
3. Gravitational waves. The existence of gravitational waves as predicted by general relativity has been indirectly proven by the shift of the period of binary systems, which is interpreted as an energy loss.

2.5 *Are there problems in this scheme?*

Beside the fact that, as we will see below, general relativity and/or quantum theory has to be modified to a theory of quantum gravity which leads to tiny deviations from standard physics, there are already three phenomena which lack any understanding within the standard theory of general relativity; see Fig. 3. These phenomena are

1. Dark matter. The rotation curves of galaxies and of galaxy clusters show that the stars far outside rotate too fast compared to the gravitational field given by the visible stars in the center of the galaxy. Also, gravitational lensing shows that there should be more matter inside galaxies and clusters of galaxies than given by visible stars. Usually this can be described or 'parametrized' by introducing additional non-visible matter, so-called 'dark matter'. Until now, no dark matter has been detected directly. The same phenomenon can, however, also be described by a modification of the law describing the generation of the gravitational field from matter in such a way that the same matter generates a stronger gravitational field than given by the ordinary Newtonian gravitational law. This explains the rotation curves as well as lensing and all other gravitational effects related to dark matter.
2. Dark energy. The accelerated expansion of the universe requires an amount of energy far beyond that given by the visible stars and the dark matter. Until now, no direct detection of dark energy has been reported.
3. The Pioneer anomaly. The two spacecrafts Pioneer 10 and 11 show, after their last flyby at Saturn and Jupiter, respectively, a constant acceleration toward the Sun that found no explanation until now. Though the accuracy is very bad, it seems that other satellites like Ulysses and Galileo also experience such an anomalous acceleration. In gravitational terms, the gravitational field generated by the Sun appears to be stronger than given by Newton's law leading to an additional acceleration toward the Sun – similar to the case of the galactic rotation curves where distant stars show an additional acceleration toward the center of the galaxy.

The question now is, whether this already might be a sign for 'new physics'? And have these effects something to do with quantum mechanics, with quantum-induced space–time fluctuations, or with the vacuum energy, for example? And can such effects be explored locally with present technologies, like interferometry?

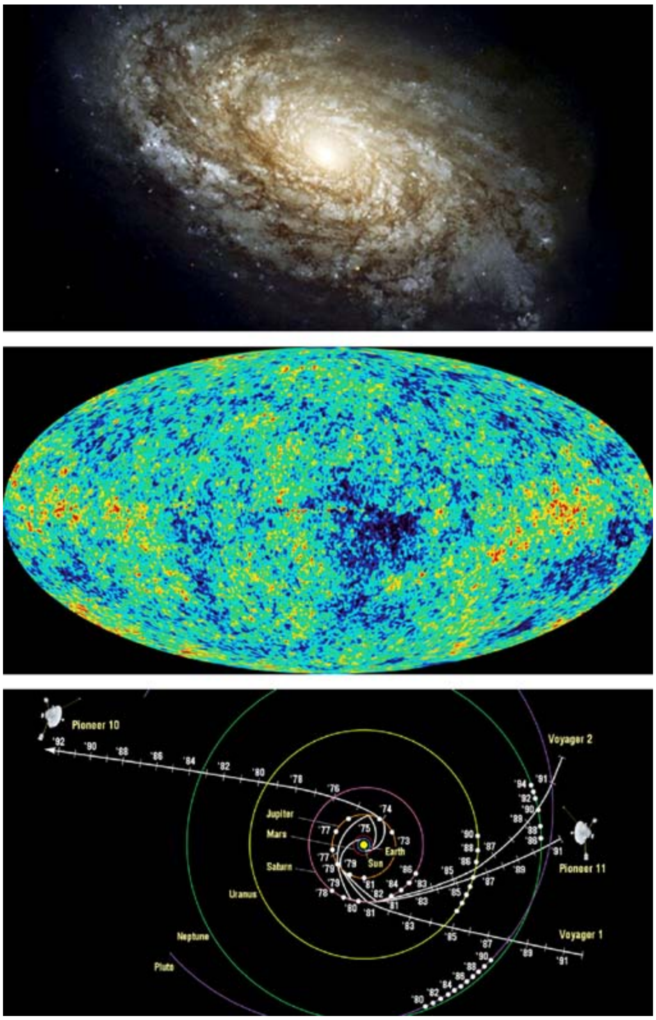

FIGURE 3 Three problems remaining unsolved within Einstein's general relativity: the rotation curves of galaxies or gravitational lensing indicating dark matter (*top*), dark energy needed for the accelerated expansion of the universe (*middle*), and the anomalous acceleration of the Pioneer spacecrafts (*bottom*)

3 The search for quantum gravity

3.1 *The need for quantum gravity*

The present status of the theoretical description of the physical world is given by four universal theories and four interactions; see Table 1. The universal theories – theories which apply to all kinds of matter and phenomena – are quantum theory, special relativity, general relativity, and statistics (or condensed matter). On the other hand, we have the four interactions – electromagnetism, gravity, and weak and strong interactions. It is one of the big wishes and hopes that there might be a true unification of all these interactions. This is not a logical necessity but it might be very useful in understanding the physical world.

On the other hand, however, there is a big problem, namely the incompatibility of quantum mechanics and general relativity. Since both theories have to be applied to all phenomena, this incompatibility necessarily has to be resolved. That theory which leads to a consistent coexistence between some kind of quantum mechanics and some kind of a theory of

Frame theories	Interactions
Quantum theory Special relativity General relativity Statistical mechanics	Electromagnetism Gravity Weak interaction Strong interaction
Problem	**Wish**
Incompatibility of quantum theory and general relativity	Unification of all interactions

TABLE 1 Frame theories and interactions

gravity is called 'quantum gravity'. Therefore, the most fundamental quest of modern physics is the search for a theory of quantum gravity.

A further incompatibility of quantum mechanics and general relativity is the problem of time: while time in quantum mechanics is an externally given parameter, in general relativity it is a dynamical variable which is influenced by the gravitational field and, thus, by the matter content in the universe:

$$\left.\begin{array}{l}\text{Time in}\\ \text{quantum theory}\\ \text{external variable}\end{array}\right\} \overset{\text{incompatible}}{\longleftrightarrow} \left\{\begin{array}{l}\text{Time in}\\ \text{general relativity}\\ \text{dynamical variable}\end{array}\right.$$

Further reasons for the need to have a quantized version of general relativity are [19]

- According to the discussion of Bohr and Rosenfeld [20], it has been shown that if matter is quantized (and this is without any doubt), then the interactions between the matter, that is, for example, the electromagnetic field, have to be quantized, too. Indeed, all attempts failed to consistently couple the classical gravitational field to quantum matter (such couplings can exist only within some approximation). In such cases superluminal velocities might occur, for example. Therefore, gravity also has to be quantized.
- The role of singularities (in particular black holes): in classical general relativity singularity theorems state that under very general assumptions singularities will occur, where all known physics will break down. Quantization of the gravitational field may circumvent the breakdown of physics in such singularities and, in particular, in the early universe and in black holes.

As a consequence, we state that there is a need for a new theory combining gravity and quantum mechanics – quantum gravity.

3.2 *The consequence*

The consequence of the need for a new theory is that this new theory, of course, has to be different from standard general relativity and quantum theory and, thus, has to violate one or more of the principles underlying these standard theories. Since, as we have shown, these standard theories are connected with the validity of the EEP, it is very likely that one or more of the principles underlying the EEP is violated in this new theory. According to what we have seen previously, a violation of the EEP necessarily reflects itself in a modification of the equations of motion, that is, in particular, of the Dirac and the Maxwell equations. As a consequence, the Einstein field equations will also be modified.

Another line of modifications is to change the notions of space and time. That means that space perhaps is no longer a continuum but, instead, shows some kind of granularity or is non-commutative or shows some other non-classical feature. Such a change in the notion again can be explored by means of the dynamics of matter only. Therefore, a change in the notions of space and time will again be manifest in modified equations of motion only. This means that we are back at the first possibility. There is no way to operationally distinguish modified notions of space and time from a modified dynamics of matter because the notions of space and time are established by the properties of the equation of motion. Only after having explored the modifications might it be a more economical description (if the modification turns out to be universal in some sense, for example) to attribute such modifications in the equations of motion to modifications of the space–time structure.

The consequence of these considerations is the task to experimentally look for modifications of the equations of motion of matter, for violations of standard physics.

3.3 *General predictions*

If we accept that there has to be some underlying theory for quantum gravity which is different from the standard theories, then we should expect at some stage deviations from the experimental outcomes as predicted by these standard theories; see the scheme of Fig. 4. These deviations are related to tiny modifications of the standard equations like the Dirac and Maxwell equations and should show up in all experiments, in particular in those experiments where particular properties of the electromagnetic field or of spin-$\frac{1}{2}$ particles are tested. These tests are mainly those tests related to tests of the principles underlying the EEP; see Fig. 5. Other searches look for a modification of the dispersion relation.

A fundamental decoherence is also one of the predictions of quantum gravity [19]. This can be searched for in interference experiments. The underlying space–time fluctuations can also be responsible for a violation of the UFF since different particles 'feel' these fluctuations in a different way; see e.g. [21].

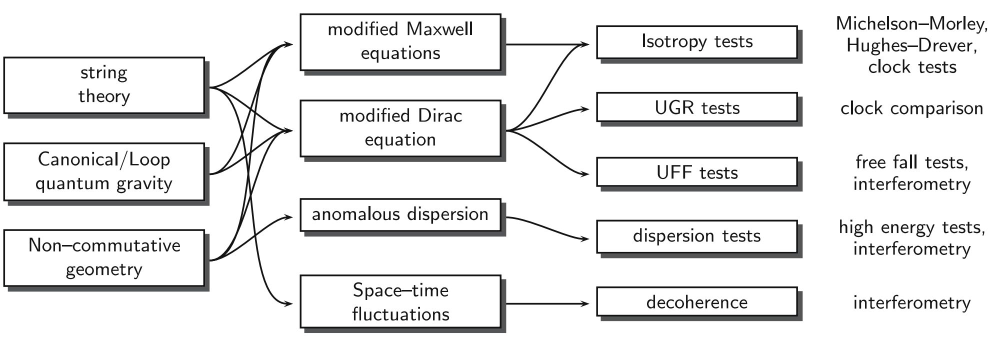

FIGURE 4 Main quantum gravity approaches and their main predictions

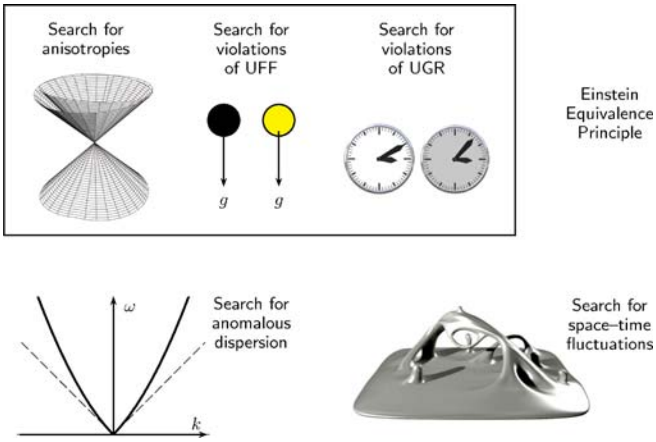

FIGURE 5 The main lines of searches for deviations from standard physics

3.4 *How to describe the search for a quantum gravity*

There is a very broad gap in the theoretical formulation of a theory of quantum gravity and the description of experiments which eventually may signal some effect which has its origin in the quantum nature of gravity. This broad gap may be bridged by a hierarchy of theories; see Fig. 6. At first, the exact formulation of quantum gravity should be – by means of renormalization and approximation schemes – turned into an effective theory, a theory which is related to at least in principle measurable quantities. It is clear that each version of a theory of quantum gravity leads to its own effective theory which is characterized by some particular terms and constants. Examples of such effective theories are the dilaton scenarios discussed by Damour, Polyakov, Piazza, and Veneziano [22–25].

On the other hand, by setting up measurable, observable quantities, one is able to formulate a huge class of phenomenological theories where each effect is characterized by some constant. These phenomenological theories can be confronted with observations which constrain the constants. The set of phenomenological theories is of course broader than the effective theories or should contain the latter as a subset. Phenomenological theories are, for example, the

Robertson–Mansouri–Sexl formalism [26–29], in which a parametrization of violations of the Lorentz transformation has been introduced (see also [30] for a gravitational modification of this approach), the

c^2 formalism [31], which parametrizes the last step in the diagram of Fig. 2, the

χ–g formalismof Ni [13], which describes breaking of LLI within the Maxwell theory, the

standard model extensionof Kostelecky and co-workers [32, 33], which describes LLI violations within the Maxwell and the Dirac theory, respectively, and, thus, includes Ni's χ–g and the c^2 formalisms, the

$TH\varepsilon\mu$ formalism [7], which generalizes the c^2 formalism to the case of gravity and, thus, includes position- and time-dependence effects, and the

PPN formalism,which has been described above [7].

Most of these phenomenological theories were invented to discuss and stress one particular effect or aspect only, but, taking all these features together, the resulting general phenomenological theory should be broader than all effective theories.

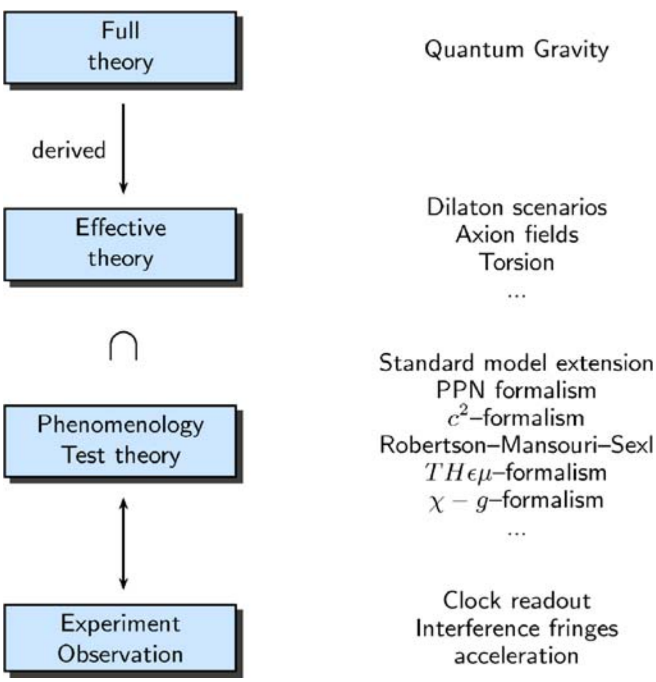

FIGURE 6 The hierarchy of descriptions of the physical world

3.5 *Phenomenology*

Since standard physics is mainly described by the Maxwell, Dirac, and Einstein equations, a phenomenology mainly consists of generalizations of these equations. There are of course infinitely many ways to generalize equations. However, for each kind of phenomenon related to the violation of one of the principles underlying the EEP, one can begin with very simple modifications. Starting from the standard Maxwell and Dirac equations (see equations (12) and (13)), these modifications may consist of introducing

- terms violating local validity of LLI without violating other principles (underlined terms in (12) and (13)),
- terms violating charge non-conservation [11] ($\chi^{\mu\varrho\sigma}$ term in (12)),
- higher derivatives which in general violate all previously violated principles (doubly underlined terms in (12) and (13)),
- non-linearities.

These modifications then yield the generalized Maxwell and Dirac equations

$$\begin{aligned} 4\pi j^\mu &= \eta^{\mu\varrho}\eta^{\nu\sigma}\partial_\nu F_{\varrho\sigma} + \underline{\chi^{\mu\varrho\nu\sigma}}\partial_\nu F_{\varrho\sigma} \\ &\quad + \chi^{\mu\varrho\sigma}F_{\varrho\sigma} + \underline{\underline{\chi^{\mu\varrho\nu\sigma\tau}}}\partial_\nu\partial_\tau F_{\varrho\sigma} \\ &\quad + \ldots + \zeta^{\mu\varrho\sigma\tau\nu}F_{\varrho\sigma}F_{\tau\nu} + \ldots\,, \qquad (12) \\ 0 &= \mathrm{i}\gamma^a D_a\psi + m\psi + \underline{M}\psi \\ &\quad + \underline{\underline{\gamma^{ab}}}D_a D_b\psi + \ldots + N(\psi)\psi\,, \qquad (13) \end{aligned}$$

with

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab} + \underline{X^{ab}}\,. \qquad (14)$$

We remark that, in most cases, the various principles are linked – as has been suggested by Schiff's conjecture. This conjecture states that the UFF also implies the EEP. Although it has been shown by Ni [13] that this is not true in a strict sense, this conjecture applies for a wide range of violations of UFF. Similarly, it can be seen from the above equations that a violation of charge conservation encoded in the tensor $\chi^{\mu\varrho\sigma}$ also implies a violation of Lorentz invariance. Therefore, as a general (but not strict) rule one can state that if something goes 'wrong' then most likely everything goes 'wrong'.

The possible effects which can be derived from the above generalized equations are

- Birefringence
- Anisotropic speed of light
- Anisotropy in quantum fields
- Violation of UFF, UGR
- Time and space variation of 'constants'
- Charge non-conservation
- Anomalous dispersion
- Decoherence, space–time fluctuations
- Modified interference
- Non-localities

In a first approach such as for example in the standard model extension, the parameters are assumed to be constant. In unification scenarios the parameters may depend on time and position through one or more fields (dilaton, cosmon). In the case of time- and/or position-dependent scalar fields the effective theory results in scalar–tensor theories.

3.6 *Search for anomalous couplings*

As far as local effects (birefringence, anisotropy, etc.) are concerned, the search for new effects is tantamount to the search for anomalous couplings. We have seen above that the EEP required a particular way of how to couple the gravitational field to the fundamental equations, that is, the Maxwell and Dirac equations. This is equivalent to using the scheme of minimal coupling of gravity to the electromagnetic and spin-$\frac{1}{2}$ fields. This means that the partial derivatives in the Maxwell and Dirac equations in special relativity have to be replaced by the corresponding covariant derivatives ('comma-goes-to-semicolon-rule' [34]). These equations locally fulfill the EEP.

However, this is only true in the lowest order of approximation. Taking quantum effects into account, the observed quantities usually behave differently because the quantum fields are extended at least over regions of the size of the Compton wavelength, and charges experience a back reaction. Indeed, in the first-order quasiclassical limit, the coupling of the spin S^μ of a Dirac particle to the curvature leads to an extra acceleration $a^\mu = \lambda_C R^\mu{}_{\nu\varrho\sigma} v^\nu S^\varrho v^\sigma$, where v^μ is the 4-velocity of the particle [35]. The corresponding acceleration is of the order 10^{-20} m/s^2. And, in [36], it has been shown that a charged particle effectively couples to the space–time curvature, $a^\mu = \alpha\lambda_C c R^\mu{}_\nu v^\nu$. Furthermore, taking into account quantum field theoretical effects the Maxwell equations also effectively couple to the space–time curvature [37]. In all cases, LLI, UFF, and UGR are violated. However, due to the smallness of the curvature these effects are too small to be detectable in solar system experiments.

An anomalous coupling of mass, charge, and spin is given by additional terms which cannot be obtained via the minimal coupling procedure. This is described best within the non-relativistic scheme using the Hamiltonian for a particle in a Newtonian gravitational field. From the Klein–Gordon or Dirac equation minimally coupled to gravity one obtains in the non-relativistic limit

$$H = \frac{\boldsymbol{p}^2}{2m} + mU(\boldsymbol{x}) , \tag{15}$$

where U is the Newtonian gravitational potential. Now we describe the various forms of anomalous couplings of the quantities mass m, spin $\boldsymbol{S}$, and charge e.

3.6.1 Mass. An anomalous coupling between mass and gravitational field can be described by means of

$$mU \to m\left(\delta_{ij} + \beta_{ij}\right) U^{ij} , \tag{16}$$

where

$$U^{ij} = G \int \frac{\varrho(\boldsymbol{x}')(\boldsymbol{x}-\boldsymbol{x}')^i(\boldsymbol{x}-\boldsymbol{x}')^j}{|\boldsymbol{x}-\boldsymbol{x}'|} \mathrm{d}^3V'$$

is the Newtonian gravitational potential tensor which fulfills $\delta_{ij}U^{ij} = U$. This leads to a violation of the UFF as described above. The tensor $m\beta_{ij}$ is called an anomalous gravitational mass and may depend on the chosen material. This can be accomplished by furthermore introducing an anomalous inertial mass tensor $m\alpha^{ij}$ which may depend on the material, too. The corresponding Hamiltonian

$$H = \frac{1}{2m}\left(\delta^{ij} - \alpha^{ij}\right) p_i p_j + m\left(\delta_{ij} + \beta_{ij}\right) U(\boldsymbol{x}) \tag{17}$$

lies at the basis of the general analysis of Haugan [38]. Applying the canonical formalism, this Hamiltonian yields the acceleration

$$a^i = \left(\delta^{ij} + \alpha^{ij}\right) \partial_j U + \delta^{ij}\beta_{kl}\partial_j U^{kl} , \tag{18}$$

which clearly shows that the anomalous mass tensors lead to violations of the UFF. For diagonal mass tensors $\alpha^{ij} = (\delta m_\mathrm{i}/m)\delta_{ij}$, $\beta_{ij} = (\delta m_\mathrm{g}/m)\delta_{ij}$ we obtain the usual form

$$a^i = \delta^{ij}\frac{m_\mathrm{g}}{m_\mathrm{i}}\partial_j U , \tag{19}$$

with the inertial and gravitational masses $m_\mathrm{i} = m + \delta m_\mathrm{i}$ and $m_\mathrm{g} = m + \delta m_\mathrm{g}$, respectively.

3.6.2 Charge. In complete analogy, an anomalous coupling of the charge to gravity can be described by adding a term κeU to the above Hamiltonian[2], where κ is a parameter of the dimension mass/charge. This gives the Hamiltonian

$$H = \frac{\boldsymbol{p}^2}{2m} + m\left(1 + \kappa\frac{e}{m}\right) U(\boldsymbol{x}) . \tag{20}$$

[2] Though charge is not at the same level as mass and spin, which are the Casimir operators of the Poincaré group, charge nevertheless is related to space–time properties through the CPT theorem. Therefore, it may be well justified to search for an anomalous coupling of charge to gravity.

In the above notation this means that we now have a charge-dependent anomalous gravitational mass. Couplings of this structure can also be found in [13, 39]. In [40] this has been generalized to the case of a charge-dependent anomalous inertial mass tensor. It has been proven in [40] that it is possible to choose parameters for electrons and protons in such a way that for neutral electrically bound systems the UFF is exactly valid, while it is violated for the charged constituents.

3.6.3 Spin. Since spin is a vector, there is a broader variability of anomalous couplings. The broad variety of these couplings has been discussed in the context of the search for a solution of the strong PC puzzle (P and C correspond to parity and charge conjugation operations; for a review, see e.g. [41, 42]). A solution is given by an axion interaction, which leads to new macroscopic forces [43]. Axions are also a candidate for dark matter in the universe; see e.g. [44]. This axion interaction leads to additional potentials between scalar and spinorial matter: mass–mass coupling

$$V(r) = -\kappa_0 \frac{\mathrm{e}^{-r/\lambda}}{r} \,, \tag{21}$$

spin–mass coupling

$$V(r) = \hbar^3 D \boldsymbol{\sigma} \cdot \widehat{\boldsymbol{r}} \left(\frac{1}{\lambda r} + \frac{1}{r^2} \right) \mathrm{e}^{-r/\lambda} \,, \tag{22}$$

and spin–spin coupling

$$\begin{aligned} V(r) = \hbar^3 T \Bigg(& (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \left(\frac{1}{\lambda^2 r} + \frac{1}{r^3} + \frac{4\pi}{3} \delta^3(r) \right) \\ & - (\boldsymbol{\sigma}_1 \cdot \widehat{\boldsymbol{r}})\, (\boldsymbol{\sigma}_2 \cdot \widehat{\boldsymbol{r}}) \left(\frac{1}{\lambda^2 r} + \frac{3}{\lambda r^2} + \frac{3}{r^3} \right) \Bigg) \mathrm{e}^{-r/\lambda} \,, \end{aligned} \tag{23}$$

where λ is the range of this potential and σ^i the usual Pauli spin matrices. D and T are in units mass^{-1}.

Spin–spin interactions of the structure (23) can arise (in the non-relativistic limit) effectively from arion couplings (see e.g. [45] and references cited therein), which are not of electromagnetic origin, or from theories with propagating torsion, where torsion is created by the elementary particle spin (see [46] or [47]). This means that even in the case that one shields all electromagnetic fields, there will be an influence of one spin on the other.

These spin couplings are encoded in the generalized Dirac equation (13). A non-relativistic limit [48, 49] leads to the generalized Pauli equation

$$\begin{aligned} \mathrm{i} \frac{\partial}{\partial t} \varphi = & -\frac{1}{2m} \left(\delta^{ij} - \alpha^{ij} - \bar{\alpha}^{ij}_k \sigma^k \right) \partial_i \partial_j \varphi + \left(c_D A^i_j + \frac{1}{m} a^i_j \right) \\ & \times \sigma^j \mathrm{i} \partial_i \varphi + \Big[m \left(U(\boldsymbol{x}) + \boldsymbol{C} \cdot \boldsymbol{\sigma}\, U(\boldsymbol{x}) + \beta_{ij} U^{ij}(\boldsymbol{x}) \right) \\ & + c_D \boldsymbol{T} \cdot \boldsymbol{\sigma} + m c_D^2 \boldsymbol{B} \cdot \boldsymbol{\sigma} \Big] \varphi \,, \end{aligned} \tag{24}$$

where α^{ij}, $\bar{\alpha}^{ij}$, A^i_j, a^i_j, $\boldsymbol{C}$, β_{ij}, $\boldsymbol{T}$, and $\boldsymbol{B}$ parametrize anomalous terms which are not present in standard special relativity and general relativity. In the classical limit this yields the acceleration for a spinning particle [48]

$$\begin{aligned} a^i = & \delta^{ij} \partial_j U(\boldsymbol{x}) + \left[\alpha^{ij} + 2 \left(\bar{\alpha}^{ij}_k + \delta^{ij} C_k \right) S^k \right] \partial_j U(\boldsymbol{x}) \\ & + \delta^{ij} \beta_{kl}\, \partial_j U^{kl}(\boldsymbol{x}) \,, \end{aligned} \tag{25}$$

which contains spin-dependent forces.

The spin-dependent interactions in the Hamiltonian can be searched for by interferometry: a polarized matter wave is split into two waves. We use a beam splitting in spin space so that half of the atoms stay in the original spin state and the other half are transferred into a state with opposite spin eigenvalue. After a time Δt the 'flipped' state is flipped back to the original state. In the meantime both states accumulate a different energy due to the interaction of the spins with external fields. This yields the phase shift

$$\delta\phi^{\mathrm{atom}} = \frac{1}{\hbar} \left[H(p, S) - H(p, -S) \right] \Delta t \,, \tag{26}$$

which generalizes the approach in [50]. The spin-dependent forces can also be detected in interferometric free fall experiments.

3.7 *Strategies for the search for quantum gravity signals*

Standard physics is experimentally well proven in standard situations, that is, for energies, velocities, distances, etc., which are well accessible on Earth. In these situations, as stated above, no single deviation from standard physics has been observed. Therefore, in order to search for deviations from standard physics one might go to non-standard situations. These situations are characterized by

- Extremely high or extremely low energies. In particular, the deviations from the standard dispersion relation for particles are negligible for standard energies (even for energies obtainable in the largest accelerators on Earth) but should influence the physics in the range of above 10^{20} eV. These energies are provided by UHECR (ultra-high-energy cosmic rays). Some hints to unusual behavior of these rays have been reported but the experimental status is still unclear. One reason for that is the gauging of the apparatuses for such high energies.
  The other extreme are extremely small energies or, equivalently, extremely low temperatures. There are speculations that one might expect new phenomena when going to lower temperatures than the lowest temperature of 500 fK achieved in Bose–Einstein condensates by the group of Ketterle [51]. See the contribution of A. Vogel et al. [52] in these proceedings.
- Extremely small or extremely large distances. It is probably no accident that the present problems in gravitational physics, that is, dark energy, dark matter, and the Pioneer anomaly are related to large distances. The Pioneer spacecrafts, for example, represent the largest-scale experiment mankind ever carried through. Large distances are also needed for the detection of ultra-low-frequency gravitational waves.
  For some time there have been speculations from higher-dimensional theories that Newton's law at very small distances (sub-mm) might be violated. (The sub-mm range is not a very small distance, but it is extremely small for the gravitational interaction.)

– Extremely short or extremely long time scales. Short time scales correspond to large energies discussed above but long time scales can be useful when searching for a signature of the influence of space–time fluctuations in physical systems. If these space–time fluctuations influence the physical system in a stochastic way, then these fluctuations should yield a $1/f$ noise which increases for small frequencies or long time scales. Therefore, ultra-stable devices like optical resonators are preferred for the search for these phenomena.

3.8 *On the magnitude of quantum gravity effects*

Since the typical laboratory energies are of the order of 1 eV and the characteristic quantum gravity energy scale is expected to be of the order of the Planck energy which is about 10^{28} eV, the quantum gravity effects in laboratory experiments are likely to come in at the order of 10^{-28} (or even lower), which appears to be well beyond the reach of laboratory experiments, in spite of the many high-precision devices which are becoming available for searching for new effects. As stressed in several recent 'quantum gravity phenomenology' reviews [1, 3, 4, 6], this suggests that the relevant phenomenology should rely on contexts of interest in astrophysics, rather than use controlled laboratory experiments. However, there may nevertheless be an important role for laboratory experiments in this phenomenology, especially considering the following observations:

– The arguments that suggest that the characteristic quantum gravity energy scale should be of order 10^{28} eV must at present be viewed as inconclusive. One really needs the correct theory, which is still not established, in order to reliably estimate this scale. It may well be that the characteristic quantum gravity energy scale is actually much lower than 10^{28} eV. For example, in scenarios with 'large extra dimensions' the quantum gravity effects, including deviations from Newton's law, would be accessible at much lower energies (see e.g. [53]). Similarly, in some string-theory-motivated 'dilaton scenarios' [24, 25] the universality of free fall (UFF) would be violated already at the 10^{-13} level, and the PPN parameter γ, which in ordinary Einsteinian gravity is exactly 1, might be different from unity by up to 10^{-5}.
– Even within the assumption that the quantum gravity effects actually do originate at the Planck scale, one can find some cases in which the small 10^{-28} effect is effectively 'amplified' [3] by some large ordinary-physics number that characterizes the laboratory setup. For example, there has been considerable work on the possibility that quantum gravity effects might significantly affect some properties of the neutral-kaon system [54, 55], and in those analyses the effects are amplified by the fact that the neutral-kaon system hosts the peculiarly small mass difference between long-lived and the short-lived kaons $M_{\mathrm{L,S}}/|M_{\mathrm{L}} - M_{\mathrm{S}}| \sim 10^{14}$.
  Similarly, in studies of quantum gravity induced interferometric noise [56] one can exploit the fact that some scenarios predict noise that increases as the frequency of analysis decreases and, therefore, searching for fundamental noises in high-precision long-term stable devices (like optical resonators) may give new access to this domain of quantum gravity effects [57].
  In an analogous way, the sensitivity of some clock-comparison experiments is amplified by a factor of $h\nu/m_{\mathrm{p}}c^2 \sim 10^{18}$, where ν is the clock frequency used and m_{p} the proton mass.
– In some rare cases the laboratory experiment actually does have the required 10^{-28} sensitivity, as illustrated by the analysis reported in [58], which studies the implications of quantum-gravity-induced wave dispersion for the most advanced modern interferometers (the interferometers whose primary intended use is the search of classical gravity waves).
– The fact that low-energy data suggest that the electroweak and strong interactions unify at the grand unified theory energy scale of $\sim 10^{16}$ GeV may encourage us to conjecture that also the gravitational interaction would be of the same strength as the other interactions at that scale. This would mean that the characteristic energy scale of quantum gravity should be some three orders of magnitude smaller than the Planck energy (see e.g. [59]).

It appears therefore that we should complement the astrophysics searches with a wide program of laboratory/controlled experiments searching for the effects predicted by some quantum gravity approaches. Although in most cases the potential sensitivity of astrophysics searches will be higher, the laboratory experiments have the advantage of providing access to a much larger variety of potential phenomena, and for the development of the relevant phenomenology it would be an extraordinary achievement to be able to analyze a new effect within a controlled/repeatable laboratory experiment with the possibility of a systematic modification of boundary and initial conditions, rather than relying on the 'one-chance' observations available in astrophysics.

3.9 *Astrophysical observations vs. laboratory experiments*

It has already been stated that one regime where one expects to observe possible quantum gravity induced effects are the high-energy cosmic rays. This makes the detection and analysis of cosmic rays to be a preferred area of the search for quantum gravity effects. However, laboratory experiments also have some advantages. Here we list and compare the advantages and disadvantages of the various approaches:

Astrophysical observations:

+ Availability of ultra-high energies of more than 10^{21} eV.
− No systematic repeatability.
− No unique interpretation: due to the lack of a systematic variation of boundary and initial conditions, it is very difficult to assign a unique cause to a certain observation. For example, since the GZK (Greisen–Zatzepin–Kuzmin) cutoff is derived from standard quantum field theory based on, for example, conventional dispersion relations and energy and momentum conservation, a violation of this cutoff may be related to a modification of the dispersion relation or to a violation of conservation laws. Only after a comparison of many data sets might it be perhaps possible to exclude some explanations.

− Very often astrophysical observations suffer from strong error bars (compared to laboratory search).

Laboratory search:

+ Repeatability of the experiment, also with a systematic variation of the initial and boundary conditions. Such a procedure can also be used to improve the quality of the result.
− The big disadvantage is that only small energies are available.
+ However, other extreme situations like ultra-low temperatures and ultra-stable devices like optical resonators can be obtained in the laboratory only.

Therefore, both astrophysical observations and laboratory experiments have their advantages and disadvantages. In fact, due to the stability and repeatability of experiments, laboratory searches for quantum gravity effects may be as promising as astrophysical observations. In the second part of this paper we show this in two examples, in the search for fundamental fluctuations in optical systems and in searches for a modified dispersion relation.

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