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Why gravity experiments are so exciting

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Abstract. Being the most fundamental interaction gravity not only describes a particular interaction between matter, but also covers issues like the notion of space and time, the role of the observer and the relativistic measurement process. Gravity is geometry and, as a consequence, allows for the existence of black holes, non-trivial topologies, a cosmological big bang, time-travel, warp drive, and other phenomena not known from non-relativistic physics. Here we present the experimental basis of General Relativity, in particular its foundations encoded in the Einstein Equivalence Principle and its predictions in the weak and strong gravity regime. We discuss various routes to search for effects possibly signaling effects of the looked for quantum theory of gravity. We lay emphasis on assumptions to be tested which are only rarely discussed in the literature like tests of Newton's axioms, tests of conservation laws, etc. We propose an experiment testing the order of time derivatives in the equation of motion.

1 Introduction – The fascination of gravity

Gravity is the most fundamental interaction in physics: it is not only a very particular interaction between particles but also is related to the notion of space and time, the description of the observer, and the relativistic measurement process. Therefore, *any issue related to gravity is also of concern for the description of all other interactions.*

But General Relativity (GR), the relativistic theory of gravity, by itself is already highly interesting. Since GR is related to the space-time geometry the gravitational interaction modifies the structure of space-time and, thus, leads to surprising phenomena like black holes. It is remarkable that there is a theory which is capable to describe that a region of space-time "disappears" and is no longer accessible to the observer. Also other unexpected effects like lensing or cosmological implications like the big bang had big impact on science, in particular on the philosophy of science, and attracted very much the general public.

At present it is very fascinating to follow the observational exploration of black holes, e.g., in the center of our milky way [1]. Accompanied by that are further mathematical studies of black hole solutions of GR found until now and also the search for new solutions of the Einstein field equations like the solution for the disk of dust [2]. There are also numerical studies of the merging of binary black holes which, when spinning, may exhibit an unexpected acceleration [3].

GR in general and solutions with black holes in particular offer very beautiful and highly interesting and stimulating mathematics studies. These studies include questions about the geometry and in particular the topology of black holes and our universe. All these issues provide a laboratory for *gedanken* experiments, e.g., leading to the information paradox [4], time travel [5] (for a recent discussion see, e.g., [6]), warp drive [7] (for a more recent discussion see, e.g., [8]), etc.

In recent years increasing effort has been spent to develop a quantum theory of gravity. A large number of people try to develop a unification of quantum theory along the lines of string theory [9], loop quantum gravity [10,11] or non-commutative geometry [12] (and references therein). While string theory lays emphasis on the particle content of our physical world and neglects somewhat the geometrical nature of gravity, loop quantum gravity starts from gravity as space-time geometry and neglects the particle content. Within string theory higher dimensional theories undergo a renaissance and, for example, black holes show up even more unusual features than known from four dimensions [13,14].

A new quantum theory of gravity also should entail new physical phenomena. Until now all experiments are in agreement with standard GR. However, big efforts are being made to find experimental signatures of quantum gravity. Any experimental result in this direction will guide the development of a quantum gravity theory. New experiments have been designed and new technologies have been developed to improve the accuracy of such experiments searching for possible quantum gravity effects. It is speculated that perhaps the LHC has the potential to see related effects.

As a consequence, gravity is the area in physics where definitively something new is expected which certainly will lead again (after GR) to a revolution of the physics paradigm. Very unusual effects are expected to happen in quantum gravity and there is a big theoretical as well as experimental effort in the search for this new upcoming theory.

Since gravity is such a fundamental interaction – it covers the notion of space-time, the space-time geometry, the observer, the measurement process, etc. – it is obvious that thinking about gravity and questioning its underlying principles opens up so many unusual possibilities which need to be tested by experiment. This ranges from questioning Newton's axioms, conservation laws, the time dependence of constants, etc. One may also speculate whether for extreme situations like extreme weak gravity, small accelerations, large accelerations, highest energies, ultralow temperatures etc. some of the principles underlying today's physics lose their meaning.

Similar exciting is quantum theory. The experimental realization of the strange behavior of quantum systems is always really astonishing, as Bohr said: "If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet". However, since quantum theory is based on a scheme which is not directly related to experiments, that is, there is no real operational approach to quantum theory, it is much more difficult to systematically questioning various assumptions underlying quantum theory. For a survey of experiments testing quantum theory see [15].

In this contribution we first describe the outstanding features of GR and then present its experimental basis. This basis consist in the principles underlying the fact that today gravity is described by a metric tensor representing the space-time geometry. This metric theory then predicts certain effects which for Einstein's GR acquires particular values. Then we give reasons why to improve these experiments and to perform new ones and present some kind of a strategy for new tests where we emphasize tests of gravity and relativity in extreme situations. Finally we focus on unusual questions related to possible effects rarely discussed in the literature like tests of Newton's axioms, of conservation laws, etc. As a matter of fact, all tests of gravity can be also regarded as a search for 'new physics'.

2 The structure of gravity

Gravity is singled out and characterized by a set of universality principles which are shared by no other interaction.

1. Universal presence of gravity
   - Gravity is *everywhere*
   - Gravity *always can be transformed away* locally
2. Universal action on masses
   - Gravity acts on *all bodies*
   - Gravity acts on all bodies *in the same way*
3. Universal action on clocks
   - Gravity acts on *all clocks*
   - Gravity acts on all clocks *in the same way*
4. Universal creation of gravitational field
   - *Each mass* creates a gravitational field
   - Each mass creates a gravitational field *in the same way*

The last of these items means that all, say, spherically symmetric masses of the *same weight* create the same gravitational field. That means that a measurement of a gravitational field only gives the mass of the gravitating body and not its composition.

3 Standard tests of the foundations of Special and General Relativity

The basic structure of GR and also of all other physics is encoded in the Einstein Equivalence Principle. This principle states that (i) if all non-gravitational interactions are switched off all pointlike particles move in the gravitational field in the same way, (ii) all non-gravitational clocks[1] are influenced by the gravitational field in the same way, and (iii) locally Special Relativity is valid, that is, all physical law are Lorentz covariant.

These principles are such important because they imply the following:

- The gravitational interaction is described by means of a metrical tensor. The mathematical frame for that is a Riemannian geometry.
- The equations of motion for a point particle, for a spin-$\frac{1}{2}$-particle, of the electromagnetic field, etc. has to be the geodesic equation, the Dirac equation, the Maxwell equations in a Riemannian space-time with a certain space-time metric.
- All these Riemannian metrics have to be the same.

Owing to its importance it is clear that these principles have to be confirmed with the highest possible accuracy. We are going to describe these experiments below.

Remark. While typical tests of quantum theory give a sinusodial function or a resonance curve, the following tests of the foundations of GR and Special Relativity (SR) are mainly null-tests. Very often this is regarded as being boring physics. Measurement of zero means no effect, no physics.

However, the meaning of these zeros become clear if one thinks at the consequences if one of these zeros turns out to be non-zero. A non-zero implies that gravity cannot be geometrized which gives a complete new understanding of gravity, a non-zero would imply a dramatic change of our conception of space and time, it would experimentally *proof* the need to develop a new fundamental theory. Such a non-zero will have great impact on the the physics of black holes, the dynamics of our universe, and also will have severe impact on technological issues like metrology, high precision navigation and positioning, etc. These zeros are at the core of our conception of the physical world.

The tests. Due to lack of space provided by the Journal we skip the description of all these experiments of the foundation of SR and GR and just refer to reviews and updates [16–22].

3.1 The consequence

The consequence of the validity of the EEP is that gravity is described by a Riemannian metric $g_{\mu\nu}$, a symmetric second rank tensor defined on a differentiable manifold which is defined as the collection of all possible physical events. The purpose of this metric is twofold: First it governs the rate of clocks, that is,

$$s = \int ds, \qquad ds = \sqrt{g_{\mu\nu} dx^\mu dx^\nu} \tag{1}$$

is the time shown by clocks where the integration is along the worldline of these clocks. Second, the metric gives the equation of motion for massive point particles as well as for light rays,

$$0 = D_v v \qquad \Leftrightarrow \qquad 0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {}^{\mu}_{\rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} \tag{2}$$

where D_v is the covariant derivative along v and $\left\{ {}^{\mu}_{\rho\sigma} \right\} = \frac{1}{2} g^{\mu\nu} \left(\partial_\rho g_{\nu\sigma} + \partial_\sigma g_{\nu\rho} - \partial_\nu g_{\rho\sigma} \right)$ is the Christoffel symbol. Here $x = x(s)$ is the worldline of the particle parametrized by its proper

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[1] Pendula and hourglasses are not allowed.

time and v the tangent vector along this worldline. While $g(v, v) = 1$ for particles, we have $g(v, v) = 0$ for light, so that we have to use some affine parameter to parametrize its wordline. More on that can be found in many textbooks on Gravity, e.g. [23–25]. It can be shown that the metric also describes the propagation of, e.g, the spin vector, $D_v S = 0$, where S is a particle spin. In generalized theories of gravity there might be additional terms in the equations of motion for v and for S.

For a general static spherically symmetric space-time metric we obtain as effective equation of motion

$$\frac{1}{2}\left(\frac{dr}{ds}\right)^2 = -\frac{1}{2}\left(\frac{E^2}{g_{tt}g_{rr}} - \frac{1}{g_{rr}}\left(1 + \frac{L^2}{r^2}\right)\right), \tag{3}$$

where E and L are the conserved energy and angular momentum, respectively. In the case of asymptotic flatness it is possible to uniquely define an effective potential [26]

$$U_{\text{eff}} = \frac{1}{2}\left(E^2 - 1 + \frac{E^2}{g_{tt}g_{rr}} + \frac{1}{g_{rr}}\left(1 + \frac{L^2}{r^2}\right)\right), \tag{4}$$

which completely governs the motion of the particle.

In order to solve the equations of motion one has to know the metric. The metric is given by some field equations

$$G = \kappa T \tag{5}$$

where G is some differential operator acting on the metric and T is the energy-momentum tensor of the matter creating the gravitational field.

4 Tests of predictions

Gravity can be explored only through its action on test particles (or test fields). Accordingly the gravitational interaction has been studies through the motion of stars, planets and satellites and of light. There are only very few experiments which observe the effects of gravity on quantum fields.

Any metric theory of gravity leads to effects like the gravitational redshift, the deflection of light, the perihelion precession, the Lense-Thirring effect, the Schiff effect, etc. GR is singled out through certain values for these effects. In the case of weak gravitational fields, a situation being present in the Solar system, and of asymptotic flatness any deviation of a gravitational theory from GR can be parametrized by a few constants, the PPN parameters [27]. Many astrophysical observations and fundamental physics space experiments are designed to make better measurement of these effects and, thus, of the PPN parameters.

For Einsteins GR we have the field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \tag{6}$$

where $R_{\mu\nu}$ and R are the Rici tensor and Ricci scalar, respectively. For a spherically symmetric gravitating body we obtain the Schwarzschild metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{1}{1 - \frac{2M}{r}}dr^2 - r^2 d\vartheta^2 - r^2\sin^2\vartheta d\varphi^2. \tag{7}$$

Use of this metric in the equation of motion yields an ordinary differential equation

$$\left(\frac{dr}{d\varphi}\right)^2 = \frac{r^4}{L^2}\left(E^2 - 1 + \frac{2M}{r} - \frac{L^2}{r^2} + 2\frac{ML^2}{r^3}\right), \tag{8}$$

which can be solved in terms of the Weierstrass $\wp$-function [28]

$$r(\varphi) = \frac{2m}{\wp(\frac{1}{2}\varphi; g_2, g_3) + \frac{1}{3}}, \tag{9}$$

where g_2 and g_3 depend on M, E, and L. This solution can be used to calculate most of the Solar system effects.

The Kerr metric is a vacuum solution of Einstein's field equation which one may describe as a rotating black hole since the metric contains parts with the product of $d\varphi\, dt$ which, when also appears for the metric of a rotating observer in Minkowski space-time. The gravitational field of a rotating star is not given by the Kerr solution. However, since for weak fields the Kerr solution is a very good approximation to the solution for a rotating star (for which no exact solution exists), one can take the Kerr solution for the description of effects related to the additional rotation term. In a weak field limit, the rotation of a star adds to the Schwarzschild metric (7) a term $J_i\, dt\, dx^i$, where J_i is the angular momentum of the rotating star.

4.1 The tests and observations

Again, due to lack of space we skip the description of the red shift, time delay, perhelion shift, Lense-Thirring effect, Schiff effect and the gravitomagnetic clock effect and refer to other reviews.

5 Why new tests?

Within the last decade it seemed that the number of high precision tests related to gravity increased considerably. This is certainly not due to some impact from the official Einstein year 2005 but is the consequence of (i) improved technology, (ii) the quest for a quantum theory of gravity, and (iii) problems in the understanding of observational data within standard GR.

5.1 Dark clouds – problems over GR

In this subsection some serious problems of GR are reported. In most cases there is no doubt concerning the data. The main problem is the interpretation of the observations and measurements. Each phenomenon which cannot be explained within standard GR is an inescapable motivation to propose new theories. However, one should nevertheless always spend a lot of effort in searching for conventional explanations. Below, beside the 'standard' interpretation of the phenomena we also mention activities regarding conventional explanations.

5.1.1 Dark matter

In 1933 it has been first been observed by Zwicky that in the Coma cluster of about 1000 galaxies the galaxies move with a velocity which is much too high compared to what one expects from the standard laws of gravity. This has been confirmed also for other galaxy clusters and also for stars in galaxies, and has been also confirmed with gravitational lensing. The gravitational field was too strong. In order to keep the Einstein equations one introduced dark matter in order to account for the strength of gravity [29]. Also structure formation needs this dark matter. However, until now there is no single observational hint at particles which could make up this dark matter. As a consequence, there are attempts to describe the same effects by a modification [30] of the gravitational field equations, e.g. of Yukawa form, or by a modification of the dynamics of particles, like the MOND ansatz [31,32], recently formulated in a relativistic frame [33]. Due to the lack of direct detection of Dark Matter particles, all those attempts are on the same footing.

Another attempt to solve the dark matter problem is to take into account th full non-linear Einstein equation. There are suggestion that at least a quite big part of the observations which usually are "explained" by dark matter can be related to a stringer gravitational field which come out while taking the full Einstein equations into account [34,35].

5.1.2 Dark energy

Observations of type Ia supernovae indicate an accelerating expansion of the universe and that 75% of the total energy density consist of a dark energy component with negative pressure [36, 37]. Furthermore WMAP measurements of the cosmic microwave background [38], the galaxy power spectrum and the Lyman-alpha forest data lines [39–41] also indicate the existence of Dark Energy, rather than a modification of the basic laws of gravitation [42]. However, also in this case there are attempts to give an explanation in terms of modified field equations, see, e.g., [43]. Recently it has been claimed that dark energy or, equivalently, the observed acceleration of the universe can be explained by inhomogeneous cosmological models, such as the spherically-symmetric Lemaitre-Tolman-Bondi model, see, e.g., [44–46].

Buchert and Ehlers [47] have shown first in a Newtonian framework that within an spatially averaging of matter and the gravitational field, rotation and shear of matter can influence the properties of the averaged gravitational field which are described in effective Friedman equations. This also holds in the relativistic case [48]. Therefore it is an open question whether dark energy is just a result of a correct averaging procedure. An influence of the averaging has been found in existing data [49,50].

5.1.3 Pioneer anomaly

The Pioneer anomaly is an anomalous unexplained acceleration of the Pioneer 10 and 11 spacecraft of

$$a_{\text{Pioneer}} = (8.74 \pm 1.33) \cdot 10^{-10}\ \text{m/s}^2 \tag{10}$$

toward the Sun [51,52]. This acceleration seems to have been turned on after the last flyby at Jupiter and Saturn and stayed constant within an 3% range. Until now no convincing explanation has been found. An anisotropy of the thermal radiation might explain the acceleration. However, while the power provided by the plutonium decay decays exponentially, the acceleration stays constant. Nevertheless, presently much work is being done on a good thermal modeling of the spacecraft. Furthermore, an anlysis of the early tracking data is on the way. Improvements of ephemerides also helps to abandon various suggested explanations and theories [53].

5.1.4 Flyby anomaly

It has been observed at various occasions that satellites after an Earth swing-by possess a significant unexplained velocity increase by a few mm/s. This unexpected and unexplained velocity increase is called the *flyby anomaly.* For a summary of recent activities, see [54]. Until now no explanation has been found but it is expected that it is either (i) a mismodeling of the thermal influence of the Earth and the Sun's radiation on the satellite, (ii) a mismodeling of reference systems, or (iii) a mismodeling of the satellite's by a point mass.

5.1.5 Increase of astronomical unit

From the analysis of radiometric measurements of distances between the Earth and the major planets including observations from Martian orbiters and landers from 1961 to 2003 a secular increase of the Astronomical Unit of approximately 10 m/cy has been reported [55] (see also the article [56] and the discussion therein). This increase cannot be explained by a time-dependent gravitational constant G because the $\dot{G}/G$ needed is larger than the restrictions obtained from LLR. Such an increase might be mimicked, e.g., by a long-term increase of the density of the Sun plasma.

5.1.6 Quadrupole and octupole anomaly

Recently an anomalous behavior of the low-l contributions to the cosmic microwave background has been reported. It has been shown that (i) there exists an alignment between the quadrupole and octupole with $>99.87\%$ C.L. [57], and (ii) that the quadrupole and octupole are aligned to Solar system ecliptic to $>99\%$ C.L. [58]. No correlation with the galactic plane has been found.

The reason for this is totally unclear. One may speculate that an unknown gravitational field within the Solar system slightly redirects the incoming cosmic microwave radiation (in the similar way as a motion with a certain velocity with respect to the rest frame of the cosmological background redirects the cosmic background radiation and leads to modifications of the dipole and quadrupole parts). Such a redirection should be more pronounced for low-l components of the radiation. It should be possible to calculate the gravitational field needed for such a redirection and then to compare that with the observational data of the Solar system and the other observed anomalies.

5.2 The search for quantum gravity

There are many experiments proving that matter has to be quantized and, in fact, all experiments in the quantum domain are in full agreement with quantum theory with all its somehow strange postulates and consequences. Consistency of the theory also requires that the quantized matter field couples with has to be quantized, too. Therefore, also the gravitational interaction has to be quantized. However, though gravity is an interaction between particles it also deforms the underlying geometry. This double-role of gravity seems to prevent the all quantization schemes to be successful in the gravitational domain.

This incompatibility of quantum mechanics and GR is not only is due to the fact that it is not possible to quantize gravity according to the known schemes but also since time plays a different role in quantum mechanics and in GR. Furthermore it is also the expectation that a quantum theory of gravity would solve the problem of th singularities appearing within GR. As a last issue, it is the wish that such a new theory also would lead to a true unification of all interactions and, thus, to a better understanding of the physical world.

Any theory is characterized by their own set of constants. It is believed that the Planck energy $E_{\mathrm{Pl}} \approx 10^{28}$ eV sets the scale of quantum gravity effects. As a consequence, all expected effects scale with this energy or the corresponding Planck length, Planck time, etc.

5.3 Possible new effects

The low energy limit of string theory as well as some quasiclassical limit of loop quantum gravity and results from non-commutative geometry suggest that many of the standard laws of physics will suffer some modifications. At a basic level these modifications show up in the equations of the standard model (Dirac equation, Maxwell equation, etc) and in Einsteins field equation. These modifications then result in

- violation of Lorentz invariance
  - different limiting velocities of different particles
  - modified dispersion relation leading to birefringence in vacuum
  - modified dispersion relation leading to frequency-dependent velocity of light in vacuum
  - orientation and velocity dependence of effects
- time- and position-dependence of constants (varying α, G, etc.)
- modified Newton-law at short and large distances.

In recent years there were increased activities to search for these possible effects. However, until now nothing has been found.

Beside these effects expected to result from quantum gravity there are some more “exotic” issues which are also worth to be tested experimentally. Such issues are (i) violations of Newtons inertial law, (ii) violation of *actio* = *reactio*, (iii) violation of charge conservation, (iv) violation of mass or energy conservation, etc. In most cases there is no basic theory from which these effects can be derived. In some cases this is due to the fact that the equations of motions cannot be derived from an action principle. Nevertheless, since these issues are at the very basis of our physical description of the physical dynamics they need to be tested to best possible accuracy.

6 How to search for "new physics"

If one looks for "new physics" then one has to measure effects which have been measured never before. One strategy where it in principle might be possible to find new things is (i) to use more precise devices, (ii) to explores new parameter regions, or (iii) to test or measure "exotic" things.

6.1 Better accuracy and sensitivity

It is clear that for a search of tiny effects a better accuracy always is a good strategy. In fact, it is amazing how the accuracy for testing Lorentz invariance, for example, increased over the last years. It took more than 20 years to improve the results of the Brillet & Hall experiment of 1979 [59] and within a few years the accuracy improved by two orders of magnitude better and is still improving further.

It might also be of interest to find examples where present day technologies have, at least in principle, the sensitivity to quantum gravity effects. One example for that are gravitational wave interferometers [60]. Todays already running gravitational wave interferometers have a strain sensitivity of 10^{-21}. With the advanced LIGO the sensitivity will become 10^{-24}. Thus, for a continuous gravitational wave with a frequency in the maximum sensitivity range between 10 and 1000 Hz a continuous observation over one year would reach a sensitivity of a bit less than 10^{-28}. This is the sensitivity needed for observing Planck scale effects (10^{28} eV) by optical laboratory devices (which have an energy scale of ~ 1 eV). This sensitivity is just the sensitivity needed to detect Planck-scale modifications in the dispersion relation for photons [60].

6.2 Extreme situations

Rather often some kind of "new physics" has been discovered when exploring new situations. We discuss various situations of this kind.

6.2.1 Extreme high energy

On possibility to explore new physics is to probe the physical processes at very high energies. One example is the LHC where in future energies of the order 10^{13} eV should be achievable. It is the hope to find signals of the Higgs particle and of supersymmetry. However, this energy range is still far away from the quantum gravity scale. The best what one can do is to observe high energy cosmic rays which have energies of up to 10^{21} eV. In fact, it has been speculated that the observations of high energy cosmic rays which according standard theories are forbidden owing to the GZK-cutoff could indicate a modified dispersion relation.

6.2.2 Extreme low energy

The other extreme, very low temperatures, might also be a tool to investigate possible signals of quantum gravity. One may speculate that the influence of possible space-time fluctuations on the dynamics of quantum systems is more pronounced for very low temperatures. One may even speculate that such space-time fluctuations may give rise to a temperature threshold above the absolute zero.

Very low temperatures may be achievable in BECs during long time of free evolution. Recently this has been achieved at the Bremen drop tower where a BEC has been created during a period of 4.7 s of free fall. It is expected that a free expansion of 1 s can be achieved soon. These BECs then may be used for novel investigations, including a search for deviations from standard predictions.

6.2.3 Large distances

The unexplained phenomena, dark matter, dark energy, and the Pioneer anomaly are related to large distances. It is discussed whether the ordinary laws of gravity are modified at large distances. Recently, some suggestions have been made:

- It has been discussed whether a Yukawa modification of the Newtonian potential may account for galactic rotations curves [30].
- In the context of higher dimensional braneworld theories also deviations from Newtons potential occurs [61]. At large distances the potential behaves with $1/r^2$, as one would expect from the Poisson equation in 5 dimensions. A comparison with cosmological and astrophysical observations has been reviewed in [62].
- From considering a running coupling constant it has been suggested that the spatial parts of the space-time metric posses a part which grows linearly with distance [63]. This approach is in agreement with present solar system tests and also describes the Pioneer anomaly [64].

6.2.4 Small accelerations

An acceleration a, being of physical dimension $\mathrm{m\,s^{-2}}$ can be related to a length scale $l_0 = c^2/a$. Now, the largest length scale in our universe is the Hubble length $L_\mathrm{H} = c/H$, where H is the Hubble constant. The corresponding acceleration is cH which order of magnitude remarkably coincides with the Pioneer acceleration and the MOND acceleration scale. As a consequence, it really seems to be mandatory to perform experiments to explore the physics for such small accelerations, see below.

6.2.5 Large accelerations

Analogously, since the smallest length scale is the Planck length l_Pl, the corresponding acceleration is $a = 2 \cdot 10^{51}\ \mathrm{m\,s^{-2}}$ which, however, is far out any experimental reach. For smaller acceleration which might be reached by electrons in the fields of strong lasers one might be able to detect Unruh radiation or to probe the physics near black holes [65,66].

6.2.6 Strong gravitational fields

Most of the observations and tests of gravity is being performed in weak fields: Solar system tests, galaxies, galaxy clusters. Recently, it became possible to observe phenomena in strong gravitational fields: in binary systems and in the vicinity of black holes.

The observation of stars in the vicinity of black holes [1] may in one or two decades give new improved measurements of the perihelion shift or of the Lense-Thirring effect. Binary systems present an even better laboratory for observing strong field effects.

The inspiraling of binary systems which has been observed with very high precision can completely explained with the loss of energy through the radiation of gravitational waves as calculated within GR [67]. The various data from such systems can be used to constrain hypothetical deviations from GR. As an example, it can be used for a test of the strong equivalence principle [68] and of preferred frame effects and conservation laws [69] in the strong field regime.

Recently, double pulsars have been detected and studied. These binary systems offer the new possibility to analyze spin effects and, thus, open up a new domain of exploration of gravity in the strong field regime [70,71]. Accordingly, the dynamics of spinning binary objects has been intensively analyzed recently [72–74].

6.3 Investigation of "exotic" issues

We describe several "unusual" questions which are posed very rarely but might also be worth to be investigated experimentally but also theoretically. One class of these strange things can be related to tests of Newton's axioms, in particular to their dynamical part related to forces:

1. Test of *actio* = *reactio*. Tests of this axiom can be encoded in the the difference of active and passive charges (electric charge, masses, magnetic moments, etc., in general each quantity which creates a corresponding field).
2. Test of the inertial law $m\ddot{\boldsymbol{x}} = \boldsymbol{F}$ where $\boldsymbol{F}$ is the force acting on a body. The question is what is being measured here? As a matter of fact, the measured acceleration together with the knowledge of the mass (which can be determined, e.g., through elastic scattering) leads to the exploration of the force. This can be illustrated with the Lorentz force. If one takes a particular type of particles, namely charged particles, and sends them through a condenser, then the trajectory will be deflected according to the voltage applied to the condenser. The deflection gives the force and the the force defines the electric field $\boldsymbol{E}$.
   Therefore, the question of testing the inertial law may have at least two meanings:
   (a) Why there are no higher time derivatives in the inertial law? (In fact, owing to back reaction all equations of motion are of higher than second order. For charged particles, for example, we have the third order Abraham-Lorentz equation. This back reaction force can be calculated from the basic equations of motion which are of second order only. Therefore, the question is why the underlying basic equations of motion are of second order.)
   (b) Does the inertial law hold for all forces, that is, does it hold if the forces become extremely large or extremely small? (In our example, do we still have $m\ddot{\boldsymbol{x}} = q\boldsymbol{E}$ even if $\boldsymbol{E}$ becomes very large or very small?)
3. Test of the superposition of forces.

6.3.1 Active and passive mass

The notion of active and passive masses and their possible non-equality has first been introduced and discussed by Bondi [75]. The *active mass* m_a is the source of the gravitational field (here we restrict to the Newtonian case with the gravitational potential U) $\Delta U = 4\pi m_\mathrm{a}\delta(\boldsymbol{x})$, whereas the *passive mass* m_p reacts to it

$$m_\mathrm{i}\ddot{\boldsymbol{x}} = m_\mathrm{p}\boldsymbol{\nabla}U(\boldsymbol{x})\,. \tag{11}$$

Here, m_i is the inertial mass and $\boldsymbol{x}$ the position of the particle. The equations of motion for a gravitationally bound two body system then are

$$m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 = Gm_{1\mathrm{p}}m_{2\mathrm{a}}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3}\,, \qquad m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 = Gm_{2\mathrm{p}}m_{1\mathrm{a}}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3}\,, \tag{12}$$

where $1, 2$ refer to the two particles and G is the gravitational constant.

For the equation of motion of the center of mass $\boldsymbol{X}$, we find

$$\ddot{\boldsymbol{X}} = \frac{m_{1\mathrm{p}}m_{2\mathrm{p}}}{M_\mathrm{i}}C_{21}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}, \qquad \text{with} \qquad C_{21} = \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} - \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} \tag{13}$$

where $M_\mathrm{i} = m_{1\mathrm{i}} + m_{2\mathrm{i}}$ and $\boldsymbol{x}$ is the relative coordinate. Thus, if $C_{21} \neq 0$ then active and passive masses are different and the center of mass shows a self-acceleration along the direction of $\boldsymbol{x}$. This is a violation of Newton's *actio* equals *reactio*. A limit has been derived by lunar laser ranging: that no self-acceleration of the moon has been observed, yields a limit of $|\bar{C}_{\mathrm{Al-Fe}}| \leq 7 \cdot 10^{-13}$ [76].

The dynamics of the relative coordinate

$$\ddot{\boldsymbol{x}} = -G\frac{m_{1\mathrm{p}}m_{2\mathrm{p}}}{m_{1\mathrm{i}}m_{2\mathrm{i}}}\left(m_1\frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} + m_2\frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}}\right)\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}\,. \tag{14}$$

has been probed in a laboratory experiment by Kreuzer [77] with the result $|C_{21}| \leq 5 \cdot 10^{-5}$.

It is interesting to note that there is no Lagrange function from which the equations of motion (12) can be derived. As a consequence, there is no Hamiltonian which means that there is no quantum version of this system. Only the equation of motion for the relative distance can be quantized.

6.3.2 Active and passive charge

Similar one can also think of active and passive charges. This has been discussed recently [78]. The resulting equations are similar to the equations for active and passive masses. The only difficulty which arises here is that the self acceleration of the center of mass cannot be observed since within atoms the timescale is too short so that this effect averages out.

However, there is one main difference to the massive case: there are positive and negative charges. This opens up the possibility to define active as well as passive neutrality. In order to exploit this possibility one has to consider a bound system in an external electric field $\boldsymbol{E}$

$$m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 = q_{1\mathrm{p}}q_{2\mathrm{a}}\frac{\boldsymbol{x}_2 - \boldsymbol{x}_1}{|\boldsymbol{x}_2 - \boldsymbol{x}_1|^3} + q_{1\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_1)\,, \qquad m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 = q_{2\mathrm{p}}q_{1\mathrm{a}}\frac{\boldsymbol{x}_1 - \boldsymbol{x}_2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|^3} + q_{2\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_2)\,, \tag{15}$$

where $q_{1\mathrm{p}}$, $q_{1\mathrm{a}}$, $q_{2\mathrm{p}}$, and $q_{2\mathrm{a}}$ are the passive and active charges. The equations of motion of the center of mass and the relative coordinate are

$$\ddot{\boldsymbol{X}} = \frac{q_{1\mathrm{p}}q_{2\mathrm{p}}}{M_\mathrm{i}}\bar{C}_{21}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} + \frac{1}{M_\mathrm{i}}\left(q_{1\mathrm{p}} + q_{2\mathrm{p}}\right)\boldsymbol{E}\,, \qquad \ddot{\boldsymbol{x}} = -\frac{1}{m_\mathrm{red}}q_{1\mathrm{p}}q_{2\mathrm{p}}\bar{D}_{21}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}\,, \tag{16}$$

where

$$\bar{C}_{21} = \frac{q_{2\mathrm{a}}}{q_{2\mathrm{p}}} - \frac{q_{1\mathrm{a}}}{q_{1\mathrm{p}}}\,, \qquad \bar{D}_{21} = \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\frac{q_{1\mathrm{a}}}{q_{1\mathrm{p}}} + \frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\frac{q_{2\mathrm{a}}}{q_{2\mathrm{p}}}\,. \tag{17}$$

Thus, if active and passive charges are different, the center of mass shows a self-acceleration along the direction of $\boldsymbol{x}$, in addition to the acceleration caused by the external field $\boldsymbol{E}$. Due to fast internal motion the self-acceleration of the center of mass is not observable.

However, it is now possible to define active neutrality through $0 = q_{\mathrm{a}1} + q_{\mathrm{a}2}$ as well as passive neutrality $0 = q_{\mathrm{p}1} + q_{\mathrm{p}2}$. Now we may prepare an actively neutral system by the condition that is creates no electric field (which may be explored by other test charges). This actively neutral system might be passively non-neutral and may react on an external electric field. Also, a passively neutral field may actively create an electric field. Only if actively neutral systems are also passively neutral, then the active and passive charge are proportional. These procedures can be carried out with high precision resulting in $\bar{C}_{12} \leq 10^{-21}$ [78]. Atomic spectra represent a cleaner test but yield only an estimate of the order $\bar{C}_{12} \leq 10^{-9}$ [78].

6.3.3 Active and passive magnetic moment

A similar analysis can be carried through with magnetic fields created by magnetic moments. If active and passive magnetic moments are different, then we again would observe a self-acceleration of the center of mass. In this case atomic spectroscopy is better and yields an estimate $\widetilde{C}_{12} \leq 10^{-5}$ [78].

6.3.4 Small accelerations

Since the effect of gravity is observed by its influence on orbits of satellites and stars, a modification of Newton's first law, $\mathrm{F} = m\mathrm{a}$ will dramatically change the interpretation of the orbits and, thus, the relation between the observation and the deduced gravitational field. This is, e.g., the basis of the MOND (MOdified Newtonian Dynamics) ansatz proposed by Milgrom [31] and put into a relativistic theory by Bekenstein [33].

The MOND ansatz replaces $m\ddot{\boldsymbol{x}} = \boldsymbol{F}$ by

$$m\ddot{\boldsymbol{x}}\mu(|\ddot{\boldsymbol{x}}|/a_0) = \boldsymbol{F}\,, \tag{18}$$

where $\mu(x)$ is a function which behaves as

$$\mu(x) = \begin{cases} 1 & \text{for} \quad |\ddot{\boldsymbol{x}}| \gg a_0 \\ x & \text{for} \quad |\ddot{\boldsymbol{x}}| \ll a_0 . \end{cases} \tag{19}$$

For Newtonian gravity this means that from the equation $\boldsymbol{F} = m\boldsymbol{\nabla} U$ we obtain the special cases

- For large accelerations or large forces: $\ddot{\boldsymbol{x}} = \boldsymbol{\nabla} U$.
- For small accelerations or small forces: $\ddot{\boldsymbol{x}}|\ddot{\boldsymbol{x}}| = a_0 \boldsymbol{\nabla} U \to |\ddot{\boldsymbol{x}}| = \sqrt{a_0 |\boldsymbol{\nabla} U|}$.

This result for small forces which are present in the outer regions of galaxies describes many galactic rotation curves very well and may also reproduce dynamics of galactic clusters. The acceleration scale a_0 is of the order $10^{-10}\ \mathrm{m\,s^{-2}}$.

A recent laboratory experiment using a torsion balance tests the relation between the force acting on a body and the resulting acceleration [79]. No deviation from Newton's inertial law has been found for accelerations down to $5 \cdot 10^{-14}\ \mathrm{m\,s^{-2}}$. However, this does not mean that the MOND hypothesis is ruled out. Within MOND it is required that the full acceleration has to be smaller than approx $10^{-10}\ \mathrm{m\,s^{-2}}$ while in the above experiment only two components of the acceleration was small while the acceleration due to the Earth attraction was still present. This means that such tests have to be performed in space. An earlier test [80] went down to accelerations of $3 \cdot 10^{-11}\ \mathrm{m\,s^{-2}}$. In both cases the applied force was non-gravitational. It might be speculated whether the MOND ansatz applies to all forces or to the gravitational force only. On Earth there is a short time and space window (of the order 1 s and 10 cm) for performing such tests [81].

Since all components of the acceleration should be such small, it is necessary to perform such tests in space. In fact it has been suggested to carry out such a test in satellites located in a Lagrange point of the Earth-Sun system.

It has also be speculated whether this MOND ansatz can describe the Pioneer anomaly [31, 52] but it has not been convincingly confirmed. In any case, it is a very remarkable coincidence the the Pioneer acceleration, the MOND characteristic acceleration as well as the cosmological acceleration are all of the same order of magnitude, $a_{\text{Pioneer}} \approx a_0 \approx cH$, where H is the Hubble constant.

6.3.5 Charge conservation

Charge conservation is a very important feature of ordinary Maxwell theory. It is

- basic for an interpretation of Maxwell-theory as $U(1)$ gauge theory, and
- it is necessary for the compatibility with standard quantum theory in the sense that it is related to the conservation of probablility.

Recently, some models which allow for a violation of charge conservation have been discussed: Within higher dimensional brane theories it has been argued that charge may escape into other dimensions [82,83] thus leading to charge non-conservation in four-dimensional space-time. Also in connection with variable-speed-of-light theories charge non-conservation may occur [84]. A very important aspect of charge non-conservation is its relation to the Einstein Equivalence Principle which is lying at the basis of General Relativity [85]. Charge non-conservation also necessarily appears if one introduces phenomenologically a mass of the photon into the Maxwell equations in a gauge-independent way [86,87].

The more important a particular feature of physics is, the more firmly this feature should be based on experimental facts. There seem to be only three classes of experiments related to charge conservation:

1. *Electron disappearing:* Charge is not conserved if electrons spontaneously disappear through $e \to \nu_e + \gamma$ or, more general, through $e \to$ any neutral particles. Decays of this kind have been searched for in high energy storage rings but nothing has been observed [88,89]. For the general process, the probability for such a process has been estimated to be $2 \cdot 10^{-22}\ \mathrm{y}^{-1}$ [88] and for two specific processes the probability is as less as $3 \cdot 10^{-26}\ \mathrm{y}^{-1}$ [89]. Even for a strict non-disappearing of electrons, the charge of electron may vary in time and thus may give rise to charge non-conservation. Therefore, while charge-conservation implies a non-disappearing of electrons, electron non-disappearing does not imply charge conservation.
2. *Equality of electron and proton charge:* Another aspect of charge conservation is the equality of the absolute value of the charge of elementary particles like electrons and protons. Tests of the equality of q_e and q_p through the neutrality of atoms [90] yield very precise estimates because a macroscopic numbers of atoms can be observed. The result is $|(q_e - q_p)/q_e| \leq 10^{-19}$.
3. *Time-variation of α:* The most direct test of charge conservation is implied by searches for a time-dependence of the fine structure constant $\alpha = q_e q_p/\hbar c$. Since different hyperfine transitions depend in a different way on the fine structure constant, a comparison of various transitions is sensitive to a variation of α. Recent comparisons of different hyperfine transitions [91] lead to $|\dot{\alpha}/\alpha| \leq 7.2 \cdot 10^{-16}\ \mathrm{s}^{-1}$. This may be translated into an estimate for charge conservation $|\dot{q}_e/q_e| \leq 3.6 \cdot 10^{-16}\ \mathrm{s}^{-1}$, provided $\hbar$ and c are constant and $q_p = q_e$. However, this cannot be done within, e.g., the frame of varying c theories.

As a consequence there seems to be *no dedicated direct experimental test of charge conservation.*

6.3.6 Test of inertial law

The question is how to test experimentally whether equations of motion have a second or higher order time derivatives. If the equation of motion is of nth order, then the solution, that is, the path depends on n initial conditions. For an equation of motion of third order we have to know the velocity and the acceleration at a certain position in order to be able to determine the path. This suggest the following experiment: We release particles at a certain position $\boldsymbol{x}_0$ with a given velocity $\boldsymbol{v}_0$ which after release move freely without any interaction (we are interested in the question whether *fundamental* equations of motion possess higher time derivatives). The essential point here is that for different particles we take different accelerations to obtain the required velocity. That is, for two particles we take different (constant, for simplicity) accelerations to reach the velocity $\boldsymbol{v}_0$ at $\boldsymbol{x}_0$. If the equation of motion is of third order, then both particles should move differently if they possess at the initial position different accelerations. If the equation of motion is of second order, then both particles should move in exactly the same way. It is not known whether such a dedicated experiment has been carried through.

As an example one may use a charged particle being accelerated by traversing a potential difference, say, a condenser. By changing the spacing between the plates of the condenser while leaving the voltage between the plates constant, the acceleration profile changes but leaves the final velocity the same. Here we assume standard energy conservation, but this is no real problem since it can be checked experimentally, too: Whether the final velocity is indeed the same can be tested, e.g., through an interference experiment directly behind the condenser. The interference pattern gives notice of the deBroglie wavelength and, thus, of the velocity (even if, due to the modifications under consideration, the relation between velocity and wavelength will also be modified, this method is still practicable: we only need *some* unique relation between velocity and wavelength – the explicit form of this relation is not needed). If the velocities, i.e. interference patterns, turn out to be always the same then our requested initial conditions are fulfilled. Then one can test the issue under consideration by doing such interference experiments at various distances from the condenser for the differently accelerated particles. This will show whether the velocities and, thus, the paths are the same for differently accelerated particles possessing the same final velocity but different final acceleration. (A similar procedure might be used in usual particle accelerators. Instead of a interferometric determination of the velocity one may also use time-of-flight measurements.)

It is easier to consider the question of the order of the time derivative on the quantum level. If on adds, e.g., a second time derivative to the Schrödinger equation, then this will change the spacing between the energy levels. A comparison with measurements yields an estimate on the strength of such a term [92]. A higher order time derivative in the Maxwell equations would, e.g., modify the dispersion relation by adding cubic or higher order energy terms. These additional terms can be observed in high energy cosmic radiation or in experiments with gravitational wave interferometers described above in section 6.1.

6.3.7 Can gravity be transformed away?

One may think that with the validity of the UFF one may eliminate gravity from the equations of motion of a neutral point particle. This is not the case. The UFF just implied that the equation of motion has to have the general form $\ddot{x}^\mu + \Gamma^\mu(x, \dot{x}) = 0$. If gravity can be transformed away then the second term has to be bilinear in the velocity $\Gamma(x, \dot{x}) = \Gamma^\mu_{\rho\sigma} \dot{x}^\rho \dot{x}^\sigma$. This is not the case in, e.g., Finsler geometries or in the example presented in [54]. These are examples where UFF is valid but Einsteins elevator fails. These examples constitute a gravity induced violation of Lorentz invariance.

7 Summary

In this contribution we described the principles underlying GR encoded in the Einstein Equivalence Principle, and their experimental verification. We also described the observations of the predictions of GR ranging from weak field Solar system test to observations of strong field effects in binary systems. Beside the standard principles we also mentioned and discussed assumptions which are usually taken for granted, though the experimental basis sometimes is of no good quality or the interpretation of the experiments is not unique. These assumptions are charge conservation, equality of active and passive mass, charge, and magnetic moment, the order of the time derivative in classical and quantum equations of motion.

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